ESTIMATING ICEBERG DRAG COEFFICIENTS USING BAYESIAN INFERENCE

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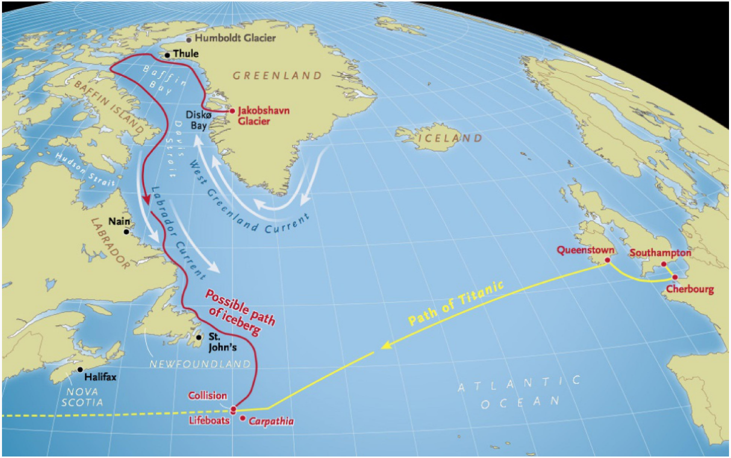
Motivation



Source: Soderman/NLSI Staff

Bayesian Inference Framework

Goal: Infer coefficients $\vec{\theta}$ given data $\vec{x}$

$$\frac{d\vec{x}}{dt} = \vec{u}$$

$$m\frac{d\vec{u}}{dt} = \vec{F}(\theta)$$

Prior: Reasonable assumptions based on past experience and knowledge.

Likelihood: A function describing the compatibility of the observed data with the model.

Posterior: Result of updating the prior given the new data.

Forward Model

$$\vec{F}(\theta) = m\vec{a} \qquad \vec{F}(\theta) = m\frac{d\vec{u}}{dt} \qquad \vec{u} = \frac{d\vec{x}}{dt}$$

Damped Harmonic Oscillator:

$$\vec{F}(\theta) = -\theta_1\vec{F}_{spring} - \theta_2\vec{F}_{damping}$$

Iceberg Model:

$$\vec{F}(\theta) = \theta_1\vec{F}_{water} + \theta_2\vec{F}_{air} + \vec{F}_{coriolis}$$

Prior & Likelihood

Prior: A distribution allowing only non-negative values for θ_1 and θ_2.

Likelihood: A function showing model-data mismatch for each given θ.

$$\pi(d|\theta) \propto exp(\mathcal{L}(\|d - G(\theta)\|^2))$$

Bayes' Rule

$$\mathit{Posterior} \propto \mathit{Prior} \cdot \mathit{Likelihood}$$

$$\pi({\color{red}\theta}|{\color{blue}d}) \propto \pi({\color{red}\theta})\pi({\color{blue}d}|{\color{red}\theta})$$

Goal: Generate samples from the posterior distribution
Method: Markov chain Monte Carlo (MCMC) sampling

Markov chain Monte Carlo (MCMC)

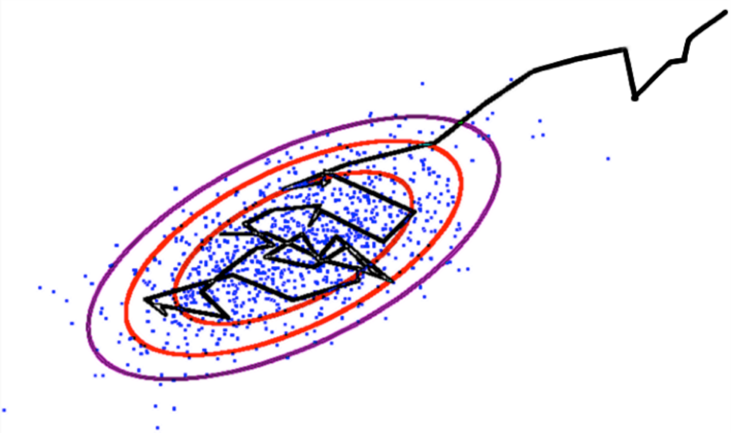
Metropolis (1953) & Hastings (1970)

θ^{t+1} only depends on θ^t

3 step algorithm (for $t = 1 \to \infty$):

1. Propose new point:
$$\hat{\theta} \sim q(\cdot|\theta^t)$$
2. Compute acceptance rate α:
$$0 \leq \alpha(\theta^t, \hat{\theta}) \leq 1$$
3. Accept / Reject:
$$\theta^{t+1} = \begin{cases} \hat{\theta} & \textit{with probability } \alpha \\ \theta^t & \textit{otherwise} \end{cases}$$

MCMC - Visualization



Source: The University of British Colombia, Ricky Chen

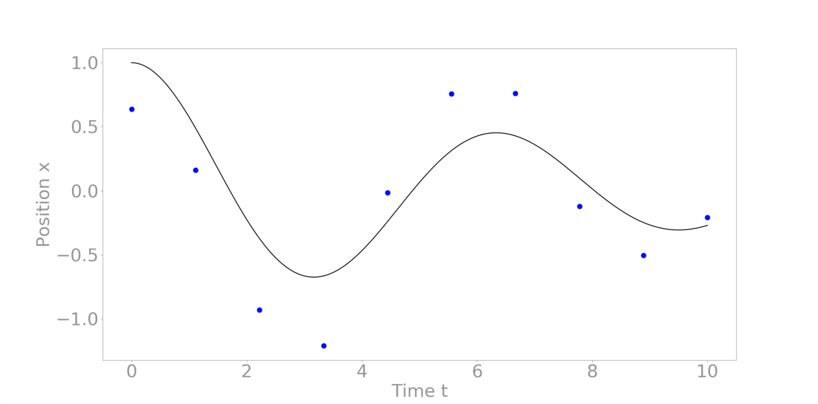
Harmonic Oscillator

Recall the forward model:

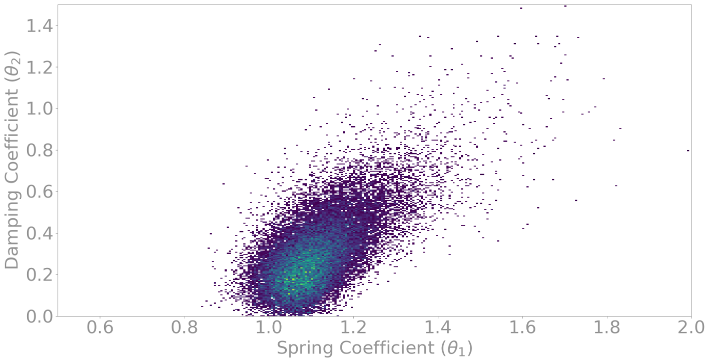
$$\frac{d\vec{x}}{dt} = \vec{u}$$

$$\vec{F}(\theta) = -\theta_1 \vec{F}_{spring} - \theta_2 \vec{F}_{damping}$$

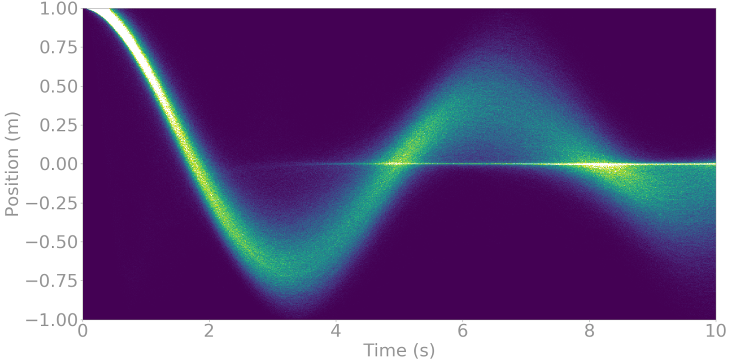
Data with Additive Noise



Sampled Posterior Distribution



Posterior Predictive Distribution



Real Iceberg Model

Recall the forward model:

$$\frac{d\vec{x}}{dt} = \vec{u}$$

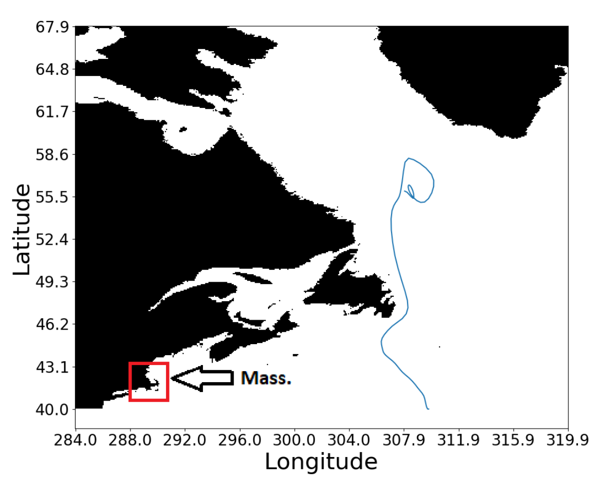
$$m\frac{d\vec{u}}{dt} = \theta_1 \vec{F}_{water} + \theta_2 \vec{F}_{air} + \vec{F}_{coriolis}$$

$$\vec{F}_{air}(x, y, t) = |\vec{v}_{air} - \vec{v}_{ice}|(\vec{v}_{air} - \vec{v}_{ice})$$

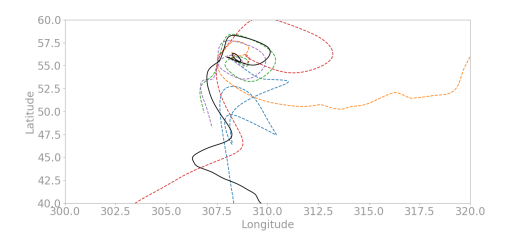
$$\vec{F}_{water}(x, y, t) = |\vec{v}_{water} - \vec{v}_{ice}|(\vec{v}_{water} - \vec{v}_{ice})$$

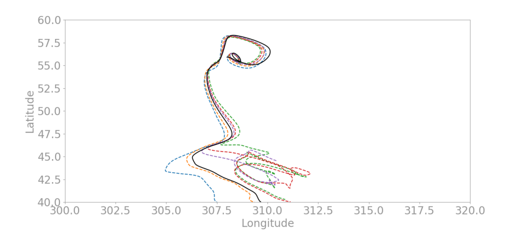
Source: Mountain(1980)

Sample Forward Model Run

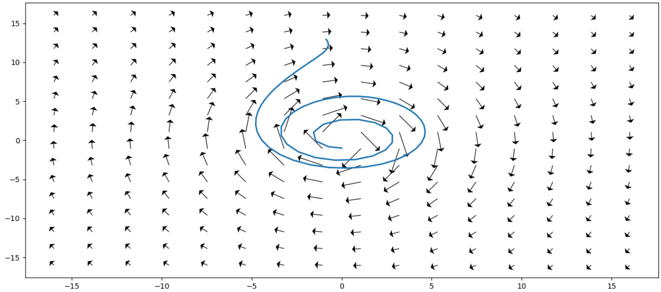


Approximate Prior v/s Posterior Predictive

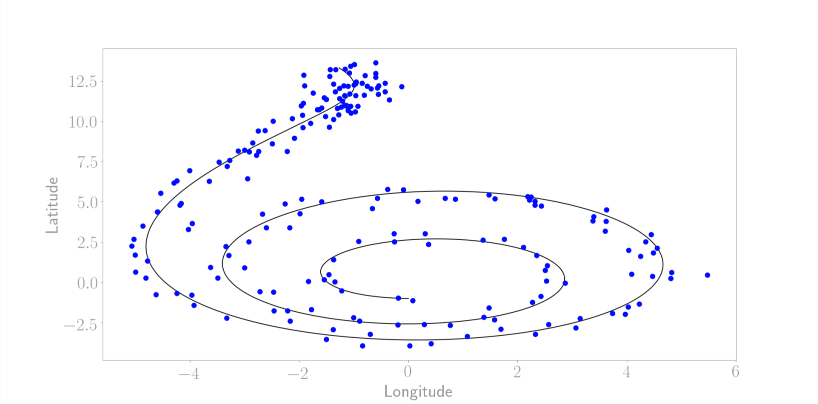




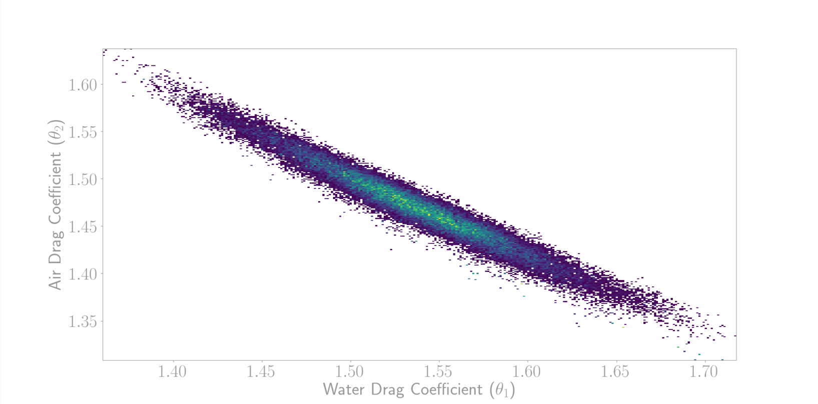
Simplified Iceberg Model



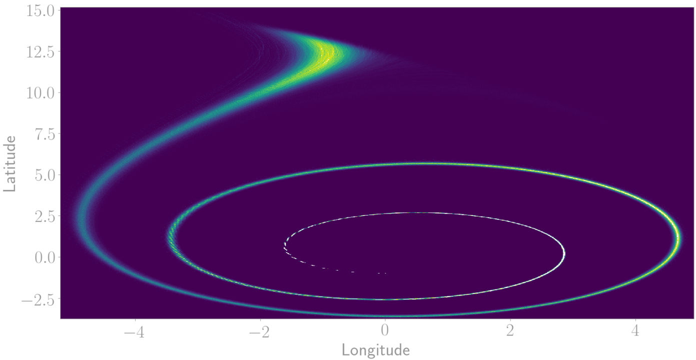
Data with Additive Noise



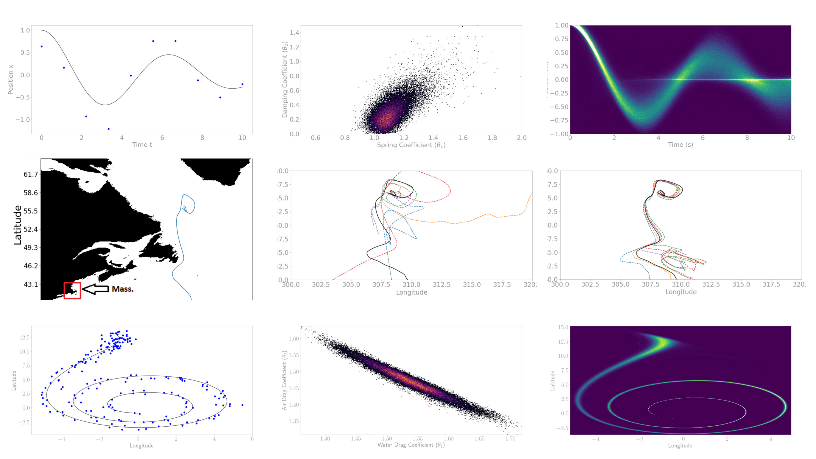
Sampled Posterior Distribution



Posterior Predictive Distribution



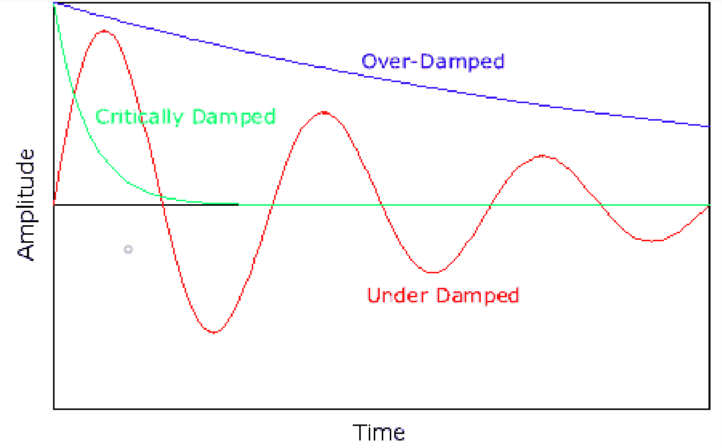
Conclusion



Position x
Time t
Damping Coefficient (θ_2)
Spring Coefficient (θ_1)
Time (s)
Latitude
Mass.
Longitude
Longitude
Latitude
Longitude
Air Drag Coefficient (θ_2)
Water Drag Coefficient (θ_1)
Latitude
Longitude

Questions

Damping Ratios of Oscillatory Systems



Source: Stuart Aitken, University of Leeds

Posterior Predictive Distribution