Investigating the Impact of Honesty in Insurance and Trading Using Game Theory Models

Dartmouth Mathematics REU

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Outline

Do you know how much money insurance fraud steals from the insurance industry every year?

Over 80 Billion Dollars!

(That's worth 50,031,269.54 MacBooks)

Objectives

We would like to build a mathematical system to better enforce honesty in the society

- Insurance fraud model: to investigate the fraudulent or honest behavior of policy holders regarding the change of the policy holders' claim amount or profit amount
- Trading model: to observe the behavior of investees with the enforcement of investment rules regarding honesty

Outline

Payoff Matrix

		PH: Fraud	PH: Honest
IN	No Inv.	$(-(s+p), p)$	$(-s, 0)$
	Inv.	$(kp-c, -(s+kp))$	$(-(c+s), \delta)$

(IN: Insurance Company, PH: Policyholder)

Model and Notations

		PH: Fraud	PH: Honest
IN	No Inv.	$(-(s+p), p)$	$(-s, 0)$
	Inv.	$(kp-c, -(s+kp))$	$(-(c+s), \delta)$

- S: honest claim amount of PH
- P: potential profit of PH if PH commits fraud
- C: cost of each secondary investigation of IN
- K: percentage on potential net fraud profit (P), which represents fine on fraudulence PH if IN realizes the fraud
- δ: benefit for PH, if IN carries out deep investigation and realizes PH is honest

Nash Equilibrium Point

From replicator equations:

$$\dot{x}_i = x_i((Ay)_i - x \cdot Ay), i = 0, ..., n$$

$$\dot{y}_j = y_j((Bx)_j - y \cdot Bx), j = 0, ..., m$$

Nash Equilibrium Point

From replicator equations:

$$\dot{x}_i = x_i((Ay)_i - x \cdot Ay), i = 0, ..., n$$

$$\dot{y}_j = y_j((Bx)_j - y \cdot Bx), j = 0, ..., m$$

x1 represents the probability that IN does not investigate

y1 represents the probability that PH tends to fraud

We obtain the equilibrium point by setting change in speed to 0:

Nash Equilibrium Point

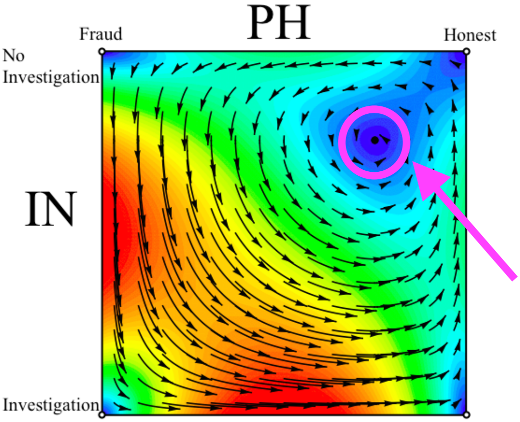
$$x_1 = \frac{kp + s + \delta s}{p + kp + s + \delta s}$$

$$y_1 = \frac{c}{p + kp + s}$$

x1 represents the probability that IN does not investigate

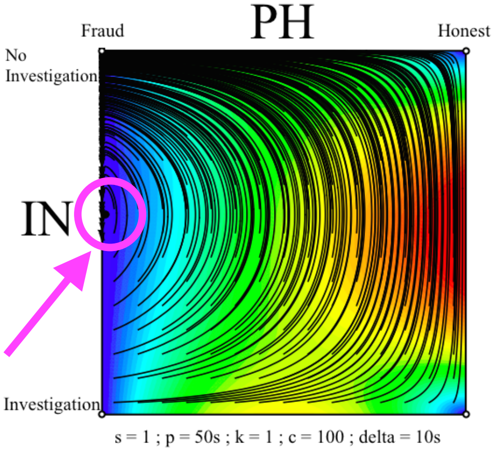
y1 represents the probability that PH tends to fraud

Dynamo 2x2



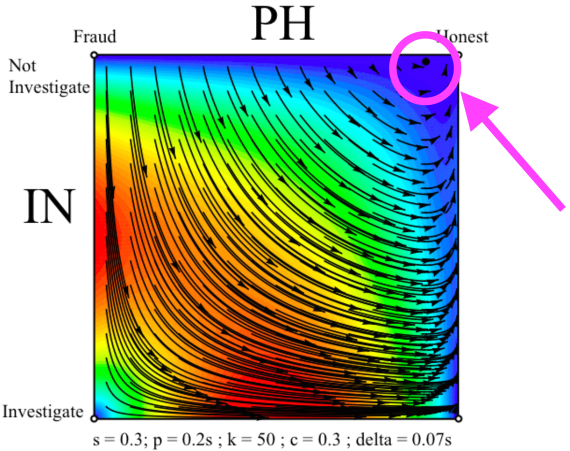
Dynamo 2x2

Nonideal



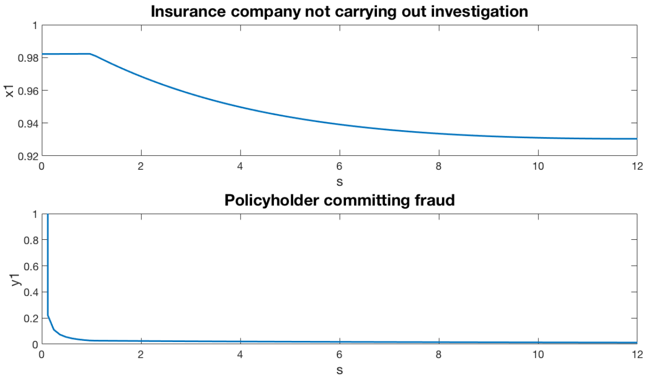
Dynamo 2x2

Ideal

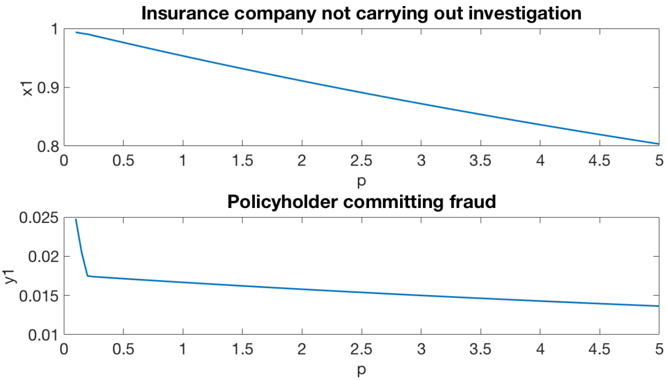


Outline

Nash Equilibrium with Change in S



Nash Equilibrium with Change in P



Outline

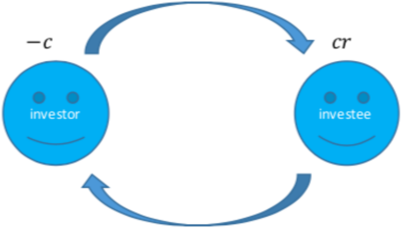
Summary

- Dishonesty tend to happen when the claim amount is small
- With the model, insurance companies can change the relevant parameters to minimize loss from insurance fraud

Outline

Basic Trust Game

Investor decides whether or not invest



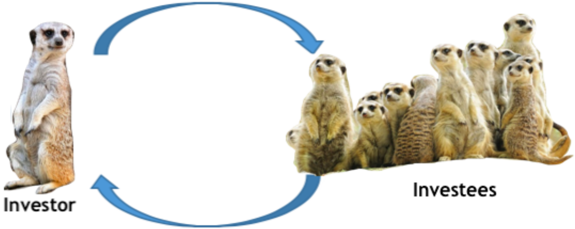
Investee decides whether or not pay back any amount to the investor

Fu, Feng
Overview and Introduction to Behavioral Analytics.
MATH 76 Lecture, 2018.

Model and Game Rules

Players

Investor decides whether or not to invest and the amount to invest in each investee



Each investee decides the percentage of revenue generated that they want to return

Model and Game Rules

Notations

- Wealth generating ability: fixed for each investee
- Flexibility: fixed for each investee
- Honesty: adjusted after every round based on investee behavior in the previous round
- Investor bank account: result
- Investee bank account: result

Model and Game Rules

Rules for Assigning Honesty Scores

- Each investee starts with a perfect honesty score
- If an investee chooses to be honest and returns promised percentage, their honesty score is not affected
- If an investee chooses to be dishonest and returns less than promised percentage, their honesty score decreases based on amount of dishonesty

Model and Game Rules

Investor's Rules for Investment

- After every round, the investor ranks investees based on the amount investees return
- The higher ranked investees, who return more to the investor, receive more investment from the investor in the next round
- If an investee has low honesty score, the investor can choose to decrease the investment amount or not invest, based on how low the honesty score is

Model and Game Rules

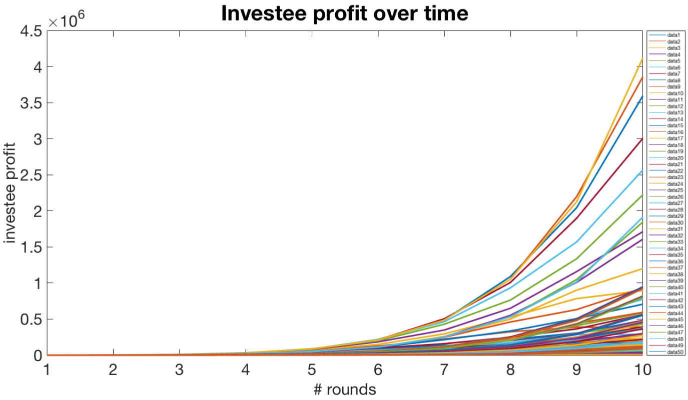
Investees' Rules for Changing Strategies

- After every round, each investee decides whether they want to copy the winning strategy from the last round
- Investees with higher flexibility scores copy the winning strategy more often than those with lower flexibility scores

Outline

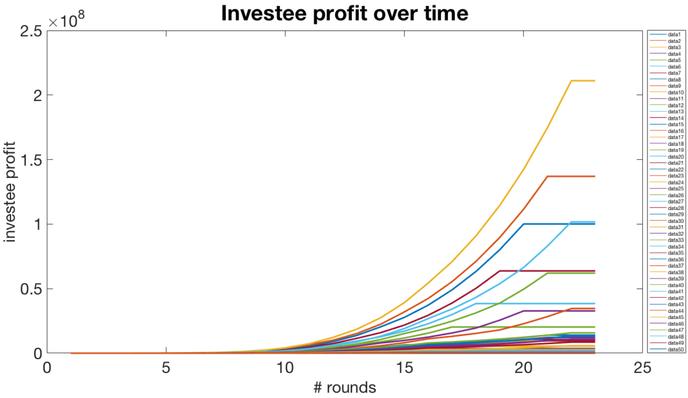
When every investee is at least slightly flexible...

After 9 rounds, every investee decides to be dishonest



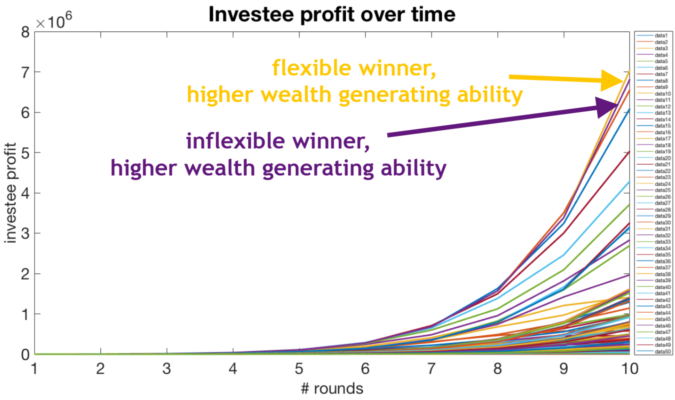
When every investee is at least slightly flexible...

After 22 rounds, the game cannot go on anymore



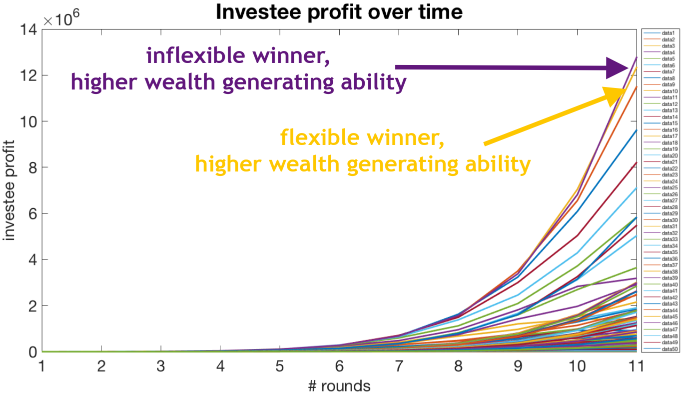
When inflexible investees are present...

After 9 rounds, every flexible investee decides to be dishonest



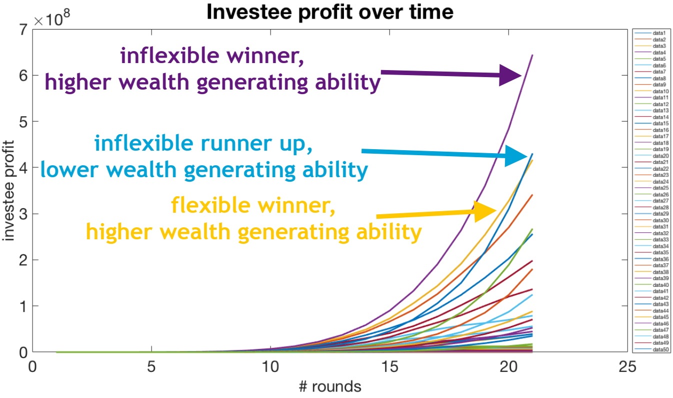
When inflexible investees are present...

After 11 rounds, flexible investees start to be honest again



When inflexible investees are present...

After 20 rounds, all investees go back to being honest and stay honest



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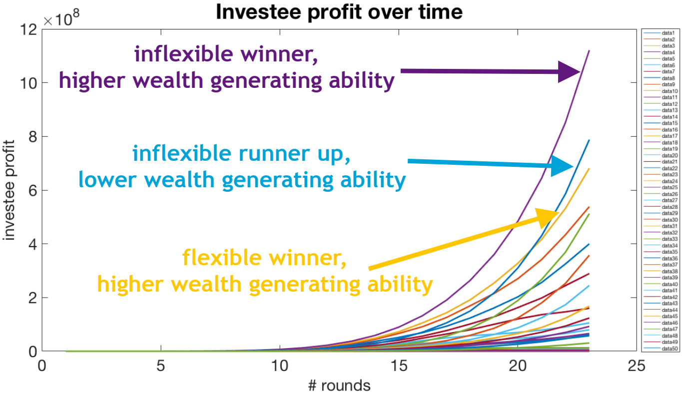
When inflexible investees are present...

After 22 rounds, the game is still going on



Outline

Summary

- Honesty is essential for the success of a society
- In the short term, dishonesty may result in more profit
- However, in the long term, honesty is important, sometimes more important than wealth generating ability, to stay successful in the competition

Future Studies

- Build honesty bots that stays perfectly honest all the time to enforce honesty in the society
- Combine the trading model with the insurance fraud model to find out how a credit system may affect the dynamics of the insurance fraud model

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- Unknown
  Bimatrix Games.

Assumptions

- Single shot game for two players, the insurance company (IN) and the policyholder (PH)
- Construct the payoff matrix from PH perspective
- IN carries out basic investigations for every claim with negligible cost before secondary investigations
- IN will and only will realize fraud of PH by carrying out secondary deep investigations
- PH profit is 0 if they only receive honest claim amount
- All profits and costs can be converted to the same unit (USD)

Game Theory

The study of Mathematical models of conflict and cooperation between intelligent rational decision-makers.

Myerson, Roger B.
Game Theory: Analysis of Conflict.
Harvard University Press, 1991.

Bimatrix Game

The values of payoff functions can be described by a bimatrix:

Player 2

	Strategy	t_1	t_2	$\dots$	t_n
	s_1	(a_{11}, b_{11})	(a_{12}, b_{12})	$\dots$	(a_{1n}, b_{1n})
Player 1	s_2	(a_{21}, b_{21})	(a_{22}, b_{22})	$\dots$	(a_{2n}, b_{2n})
	$\vdots$	$\dots$	$\dots$	$\dots$	$\dots$
	s_m	(a_{m1}, b_{m1})	(a_{m2}, b_{m2})	$\dots$	(a_{mn}, b_{mn})

The values of payoff functions can be given separately for particular players:

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}, \qquad B = \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ \dots & \dots & \dots & \dots \\ b_{m1} & b_{m2} & \dots & b_{mn} \end{pmatrix}.$$

Unknown
Bimatrix Games.