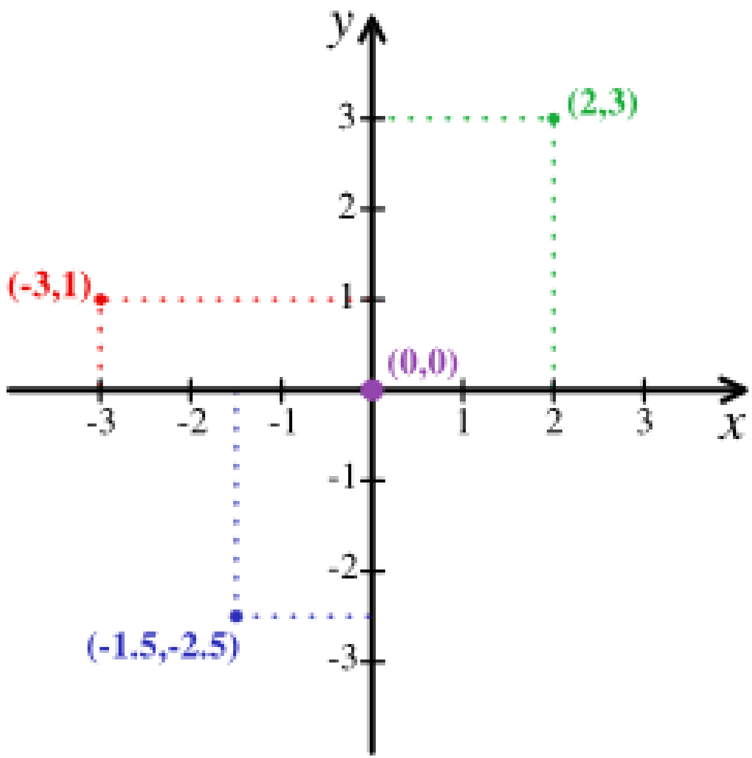
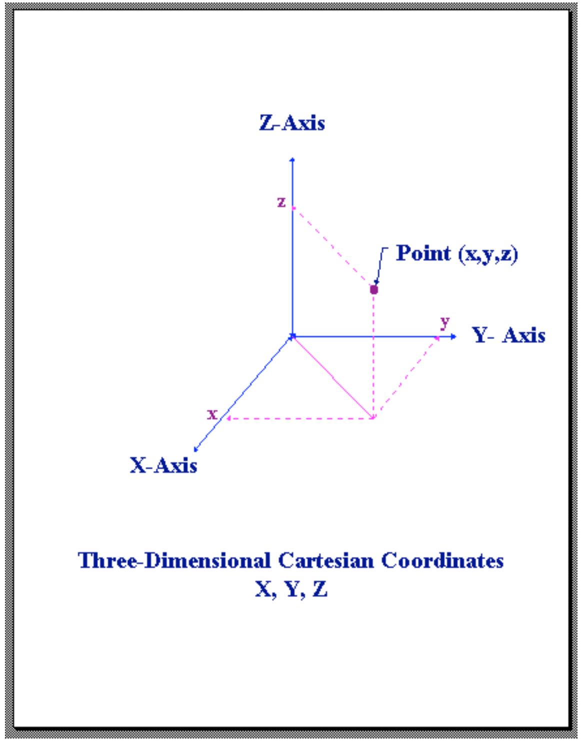
Introduction to coordinates and vectors

October 29, 2007

Cartesian (Rectangular) Coordinates system





Distance between two points

Let $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ be two points in three dimensions, the **distance** between the points is

$$|P_1P_2| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

Equation of a sphere

The equation of a sphere with center $C(h, k, l)$ and radius r is

$$(x-h)^2 + (y-k)^2 + (z-l)^2 = r^2$$

Constant planes

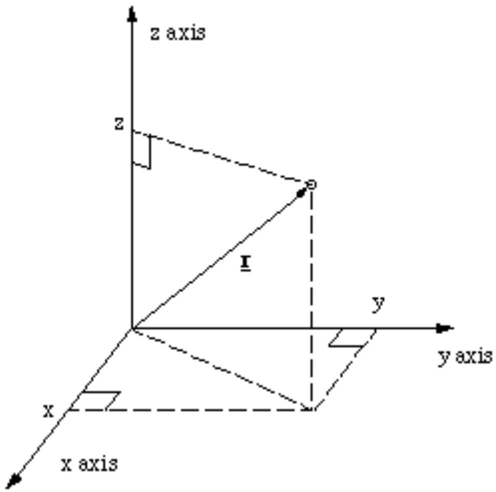
Let a, b, c be any constants, then

$x = a$ is a plane parallel to the yz-plane.

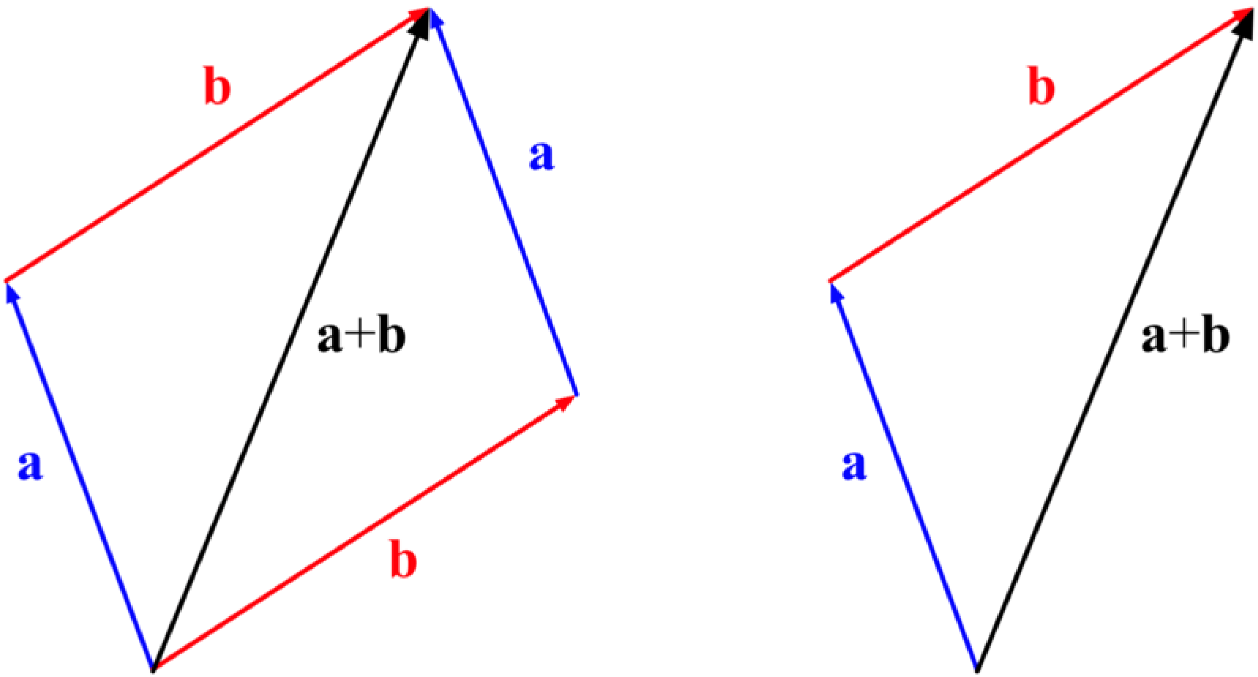
$y = b$ is a plane parallel to the xz-plane.

$z = c$ is a plane parallel to the xy-plane.

Position Vector and Displacement Vector



Vector Addition - Geometrically



Vectors in the vector space $\mathbb{R}^n$

In this class a **scalar** is simply a real number. An element in $\mathbb{R}$.

A **vector** in $\mathbb{R}^2$ is a pair $\langle x, y \rangle$ of real numbers.

A **vector** in $\mathbb{R}^3$ is a triple $\langle x, y, z \rangle$ of real numbers.

A **vector** in $\mathbb{R}^n$ is an n-tuple $\langle x_1, x_2, \ldots, x_n \rangle$ of n real numbers.

Operations on vectors - Algebraically

Vector Addition: Let $\mathbf{a} = \langle a_1, a_2, \ldots, a_n \rangle$ and $\mathbf{b} = \langle b_1, b_2, \ldots, b_n \rangle$ in $\mathbb{R}^n$ then their **sum** is

$$\mathbf{a} + \mathbf{b} = \langle a_1 + b_1, a_2 + b_2, \ldots, a_n + b_n \rangle$$

Scalar Multiplication: Let $\mathbf{a} = \langle a_1, a_2, \ldots, a_n \rangle$ be a vector in $\mathbb{R}^n$ and k any scalar then

$$k\mathbf{a} = \langle ka_1, ka_2, \ldots, ka_n \rangle$$

Vector between two points

Let $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ be two points, the vector from A to B is

$$\vec{AB} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$$

Length of a vector

For a vector $\mathbf{a} \in \mathbb{R}^2$, $\mathbf{a} = \langle a_1, a_2 \rangle$

$$|\mathbf{a}| = \sqrt{a_1^2 + a_2^2}$$

For a vector $\mathbf{a} \in \mathbb{R}^3$, $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$

$$|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

The standard basis vectors

The standard basis vectors in $\mathbb{R}^2$ are
$\mathbf{i} = \langle 1, 0 \rangle$ and $\mathbf{j} = \langle 0, 1 \rangle$.

The standard basis vectors in $\mathbb{R}^3$ are
$\mathbf{i} = \langle 1, 0, 0 \rangle$ and $\mathbf{j} = \langle 0, 1, 0 \rangle$ and $\mathbf{k} = \langle 0, 0, 1 \rangle$.

The standard basis vectors in $\mathbb{R}^n$ are
$\mathbf{e}_1 = \langle 1, 0, \ldots, 0 \rangle$, $\mathbf{e}_2 = \langle 0, 1, 0, \ldots, 0 \rangle, \ldots,$
$\mathbf{e}_n = \langle 0, \ldots, 0, 1 \rangle$.