

# Math 9 Fall 2003

## Homework project II (20 points)

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### Directional Derivatives and Differentiable Functions

Due date Monday, November 24, at the end of the lecture

## Description of the Project

Let  $f(x, y) : \mathbb{R}^2 \rightarrow \mathbb{R}$  be a function defined as follows:

$$(1) \quad f(x, y) = \begin{cases} 0 & \text{for } (x, y) = (0, 0); \\ \frac{5xy^2}{x^2+y^2} & \text{for } (x, y) \neq (0, 0). \end{cases}$$

- (1) Prove that  $f$  defined this way is a continuous function on the whole real plane  $\mathbb{R}^2$ .
- (2) Let  $\mathbf{u} = \langle \frac{a}{\sqrt{a^2+b^2}}, \frac{b}{\sqrt{a^2+b^2}} \rangle$  be a unit vector. Find the value of the directional derivative  $D_{\mathbf{u}}f(0, 0)$  as a function of  $a$  and  $b$ .
- (3) Find the gradient  $\nabla f(0, 0)$ .
- (4) Show that  $D_{\mathbf{u}}f(0, 0) \neq \mathbf{u} \cdot \nabla f(0, 0)$ , for some vectors  $\mathbf{u}$ .
- (5) Use Theorem 15.6.3 to conclude that the function  $f(x, y)$  is not differentiable at the point  $(0, 0)$  despite the fact that it is continuous on  $\mathbb{R}^2$  and has all the directional derivatives at  $(0, 0)$ .

**Hint:** You can not conclude that  $f$  is continuous at  $(0, 0)$  as a ratio of two continuous functions, since the denominator is zero at  $(0, 0)$ . For the same reasons you can not differentiate the function with respect to  $x$  and  $y$  variables at  $(0, 0)$  in the straightforward way. Instead use Definitions 15.2.1, 15.2.4, 15.3.4, 15.6.2, and 15.6.3 from the Stewart "Calculus" textbook.

## REGULATIONS CONCERNING THIS PROJECT

- 1: Students are encouraged to work in groups of 3 or 4. **No group may consist of more than 4 students.** Students from different sections of Math 9 may work together in a single group.
- 2: Each group must submit **exactly one written report** to exactly one of the Math 9 instructors. Each student in a group is responsible for understanding everything in the written report and should be ready to present it to a professor if asked. **Write the names of all the students from the group and their instructors on the first page of the project.**
- 3: **The report must be written in claim-proof form and in full sentences.** (See "proof-writing suggestions" on the Math 9 homework assignments page.) Correct answers without proper justification are not acceptable.
- 4: Students will collaborate within one group, **but must not collaborate with members of other groups.**
- 5: Students are encouraged to consult mathematical textbooks and Math 9 instructors.
- 6: **The project is due on Monday November 24.** Each group must designate one member to submit the written report at **the beginning of that member's Math 9 lecture.**