Math 8 - Winter 2019
Practice Exam III

Your name: ______________________________

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INSTRUCTIONS

This is a closed book, closed notes exam.

You have 3 hours.

Use of calculators is not permitted.

(1) Given the function

$$f(x) = e^x \cdot \sin(x).$$

a) Find the Taylor polynomial $T_3(x)$ of degree 3 of $f(x)$ at $x = 0$.

b) Use your result from part a) to evaluate the limit

$$\lim_{x \to 0} \frac{e^x \cdot \sin(x) - x}{3x^2}$$

(2) Find the radius of convergence of the series

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}\, x^{2n}}{4^n}.$$

(3) Consider the planes

$$E_1 : x + 2y + 3z = 5 \quad \text{and} \quad E_2 : 2x + y + z = 4.$$

Find the line of intersection of the two planes.

(4) Find the arclength $\ell(c)$ of the curve

$$c: \ \mathbf{r}(t) = \left\langle e^{t+1}\cos(t), 4, e^{t+1}\sin(t) \right\rangle \quad \text{where} \quad -1 \le t \le 1.$$

(5) A constant force $F = \langle 2, 1, 4 \rangle$ moves an object along the line segment from $(1, 0, 2)$ to $(2, 7, 6)$. Find the work done by the force.
Note: Here distance is measured in feet and force in pounds.

(6) Let f be the function given by

$$f(x,y) = 4 + y + \frac{1}{x}$$

a) Find the domain of the function f.

b) Find the equation of the level curves $L_f(k)$ defined by $f(x,y) = k$ for $k = -1, 0, 1$ and sketch these curves.

c) Sketch the gradient vector $\mathbf{grad}(f)(x, y) = \nabla f(x, y)$ at several points along each of the level curves in part b). Be sure your sketch shows clearly the direction in which the gradient points and its relationship to the level set.

d) Find the tangent line to the level set $f(x, y) = 1$ at the point $P = (-4, 1)$.

(7) Recall that if x and y are the lengths of two sides of a triangle and θ is the angle between these two sides, then the area of the triangle is given by

$$A(x, y, \theta) = \frac{1}{2}xy\sin(\theta).$$

Suppose that at a certain instant, the two sides have lengths 4 and 8 and are increasing at the rate of $\frac{1}{10}$ in/sec and $\frac{2}{10}$ in/sec and the angle between them is $\frac{\pi}{6}$ and is decreasing at the rate of $\frac{1}{10}$ radians/sec.

At what rate is the area changing at that instant?

(8) a) Find the tangent plane T_P of the function

$$f(x,y) = \cos(xy) - y^3 \text{ at the point } P = (\frac{\pi}{2}, 1, f(\frac{\pi}{2}, 1))$$

b) Find the line L passing through $Q = (1,2,0)$ which is orthogonal to the tangent plane T_P.

(9) a) Find the rate of change of the function

$$f(x,y) = \sqrt{10 - x^2 - 2y^2} \quad \text{at the point} \quad P = (2, -1)$$

in the direction to the point $Q = (4, 1)$.

b) Determine the maximal rate of change of f at P and the direction in which it occurs.

(10) Let f be the function defined by

$$f(x, y) = -2x + y - xy + 1 \quad \text{in the region} \quad D : x^2 - 2 \leq y \leq 2.$$

Find the absolute maxima and minima of f in D.

(11) Use Lagrange multipliers to find the maximum volume of a rectangular box with sides parallel to the coordinate axes that can be inscribed in the ellipsoid

$$D : 2x^2 + y^2 + 3z^2 = 3.$$

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