# Math 8 Winter 2019, Practice Exam II

**Free Response.** *On each question, except the multiple choice question 12, you must justify your answers.*

(1) Let $f$ be the function

$$f(x, y) = xe^y + \ln(y) + \frac{1}{x^2y + 1}.$$

Compute the partial derivatives $f_x$ , $f_y$ and $f_{yy}$.

(2) Find the area of the triangle with vertices

$$A = (1,1,1)\,,\ B = (2,4,5) \ \text{ and } \ C = (3,1,2).$$

(3) Find parametric equations for the line through the point $P = (1, 1, 1)$, perpendicular to the line

$$L : x = 1 + t,\ y = 1 + 2t,\ z = 1 + 3t$$

and parallel to the $xy$-plane (i.e., to the plane $z = 0$).

(4) (a) Check that the lines

$$L_1: \; x = 1 + t, y = 1 + 2t, z = 2 + 3t$$

and

$$L_2: \; x = 4 - t, y = 1 + tz = 1 + 2t$$

intersect.

(b) Find the equation of the plane that contains the two lines in part (a).

(5) For the following two vectors, find the scalar and vector projection of $\mathbf{u}$ onto $\mathbf{v}$. Then write $\mathbf{u}$ as the sum of a vector parallel to $\mathbf{v}$ and a vector orthogonal to $\mathbf{v}$.

$$\mathbf{u} = \langle 5, 1, 5 \rangle \quad \text{and} \quad \mathbf{v} = \langle 3, 0, 4 \rangle.$$

(6) A woman exerts a horizontal force of 25 pounds as she pushes a box up a ramp that is 10 feet long and inclined at an angle of 30 degrees above the horizontal.

Draw a picture of the situation then find the work done by the woman on the box.

(7) Give parametric equations for the curve of intersection of the surfaces

$$S_1 : \ x^2 + \frac{z^2}{4} = 1 \quad \text{and} \quad S_2 : \ y = xz.$$

(8) Find the arclength $\ell(c)$ of the curve

$$c: \ \mathbf{r}(t) = \left\langle t^2\cos(t), \frac{t^3}{3}, t^2\sin(t) \right\rangle \quad \text{where} \ \ 0 \leq t \leq 2.$$

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(9) Let $c$ be a curve given by

$$c: \ \mathbf{r}(t) = \left\langle t^3, \frac{1}{1+t}, t^2 + t \right\rangle \quad \text{where} \quad t \in [0, 5].$$

Find a parametrization of the tangent line of the curve at the point $P = (8, \frac{1}{3}, 6)$.

(10) Find the domain of the function

$$f(x, y) = \ln(x^2 + y^2 - 1).$$

Then sketch both the graph of the function and the contour map. In graphing the contour map, show several level curves and label them according to the value of the function.

(11) Compute the following limits or show that they do not exist.

a) $\lim_{(x,y)\to(0,0)} \frac{x^2 y}{3x^2 + y^2}$.

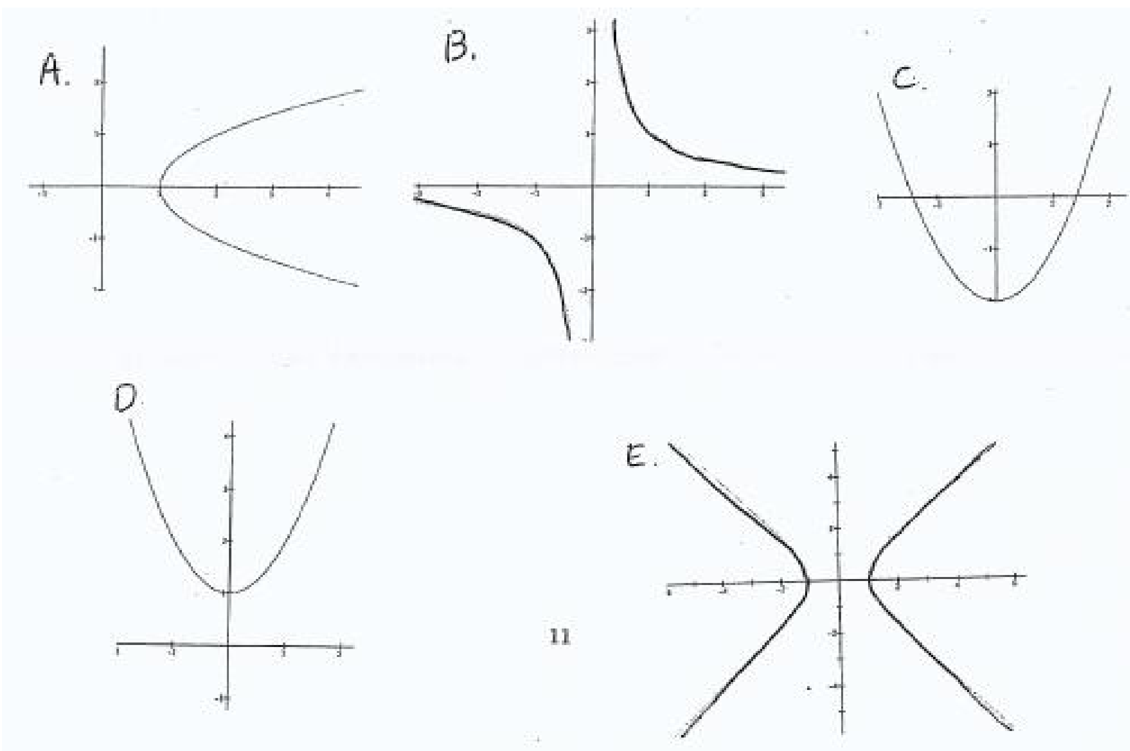
b) $\lim_{(x,y)\to(0,0)} \frac{4y^2}{y^2 + x^4}$.

(12) For each of the functions below, exactly one of the curves labelled A to E is a level curve of the function.
Fill in the blank next to the function with the letter of its level curve. **Note:** A level curve may have more than one branch, as a hyperbola does for example.

_______ $f(x,y) = \ln(xy)$

_______ $f(x,y) = \exp(\frac{x}{y^2+1})$

_______ $f(x,y) = \sqrt{y - x^2}$



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