Math 8: Calculus in one and several variables
Winter 2019 - Homework 6

Return date: Friday 02/15/19

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keywords: *derivatives along curves, functions of several variables*

Instructions: Write your answers neatly and clearly on straight-edged paper, use complete sentences and label any diagrams. Please show your work; no credit is given for solutions without work or justification.

exercise 1. *(5 points)* Consider the curve given by

$$\mathbf{r}(t) = \langle t^2, t^3 + 3, t + 1 \rangle$$

a) Find all points on the curve for which the tangent vector is parallel to the plane $E : 2x + y + z = 7$.

b) Find a parametrization of the tangent line of $\mathbf{r}(t)$ at $t = 1$.

c) Find the unit tangent vector to $\mathbf{r}(t)$ at $t = 1$.

exercise 2. *(3 points)* Find the arclength $\ell(c)$ of the curve

$$c : \mathbf{r}(t) = \left\langle \frac{t^2}{2}, \frac{2\sqrt{2}}{3} t^{3/2}, t \right\rangle \quad \text{where} \quad 0 \leq t \leq 2.$$

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exercise 3. *(6 points)* Find the domain of the following functions, then sketch the graphs and several level curves:

a) $f(x, y) = 10 - x - y$.

Suggestion for graphing: Just sketch the part of the graph in the first octant.

b) $f(x, y) = \sqrt{4x^2 + y^2}$.

exercise 4. *(4 points)* Consider the function $f(x, y) = \frac{x^2+y^2}{2x}$. Sketch several level curves $f(x, y) = k$, choosing at least two positive values of k and at least two negative values of k.

Suggestion: After writing $f(x, y) = k$, clear the denominator, bring everything to one side of the equation and complete the square in x.

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exercise 5. *(2 points)* Compute the following limits or show that they do not exist.

a) *(0 points)* $\displaystyle\lim_{(x,y)\to(0,0)} \frac{e^{x^2+y^2}-1}{x^2+y^2}$.

Hint: Look at the Maclaurin series of e^t.
This problem is **optional**.

b) *(2 points)* $\displaystyle\lim_{(x,y)\to(0,0)} \frac{3xy^2}{y^4+x^2}$.

Hint: Look at the curves $\mathbf{r}_1(t) = (t, 0)$ and $\mathbf{r}_2(t) = (t^2, t)$.