

Math 8: Calculus in one and several variables
Winter 2019 - Homework 4

Return date: Friday 02/01/19

keywords: *dot product, work, orthogonal projections, cross product*

Instructions: Write your answers neatly and clearly on straight-edged paper, use complete sentences and label any diagrams. Please show your work; no credit is given for solutions without work or justification.

exercise 1. (*4 points*) Consider the triangle with vertices

$$A = (0, 1, 3), B = (1, 2, 4) \text{ and } C = (2, 4, 1).$$

- a) Find the cosine of the angle at vertex A .
- b) Is the triangle a right-angled triangle? If so, at which vertex does the right angle occur?

exercise 2. (*3 points*) A sled is pulled along a level path through the snow by a rope. A 40-lb force acting at an angle of 30° above the horizontal moves the sled 100 ft. Find the work done by the force.

exercise 3. (*2 points*) Let \mathbf{u} and \mathbf{v} be vectors.

- a) Use the geometric interpretation of the dot product to show that

$$|\mathbf{u} \bullet \mathbf{v}| \leq |\mathbf{u}| |\mathbf{v}|.$$

Note: This inequality is called the Cauchy-Schwarz inequality.

- b) Can $|\text{proj}_{\mathbf{v}}(\mathbf{u})|$ be greater than $|\mathbf{u}|$? Explain your answer.

exercise 4. (*4 points*) Find the scalar and vector projection of \mathbf{u} onto \mathbf{v} . Then write \mathbf{u} as the sum of a vector parallel to \mathbf{v} and a vector orthogonal to \mathbf{v} .

- a) $\mathbf{u} = \langle -1, 4, 8 \rangle$ and $\mathbf{v} = \langle 12, 1, 2 \rangle$.
- b) $\mathbf{u} = \langle 1, 4 \rangle$ and $\mathbf{v} = \langle 2, 3 \rangle$.

exercise 5. (*4 points*) For each of the following vectors, find the cross product $\mathbf{u} \times \mathbf{v}$ and verify that it is orthogonal to both \mathbf{u} and \mathbf{v} :

- a) $\mathbf{u} = \langle 4, 3, -2 \rangle$ and $\mathbf{v} = \langle 2, -1, 1 \rangle$.
 - b) $\mathbf{u} = t\mathbf{i} + \cos(t)\mathbf{j} + \sin(t)\mathbf{k}$ and $\mathbf{v} = \mathbf{i} - \sin(t)\mathbf{j} + \cos(t)\mathbf{k}$.
-

Math 8: Calculus in one and several variables
Winter 2019 - Homework 4

Return date: Friday 02/01/19

exercise 6. (*3 points*)

a) Find the area of the parallelogram with vertices

$$A = (1, 0, 2), B = (3, 3, 3), C = (7, 5, 8) \text{ and } D = (5, 2, 7).$$

b) Find the area of the triangle with vertices A , B and C , where A , B and C are the points given in part a).
