Math 8: Calculus in one and several variables
Winter 2019 - Homework 3

Return date: Friday 01/25/19

keywords: *limits and integrals with Taylor series, vectors, dot product*

Instructions: Write your answers neatly and clearly on straight-edged paper, use complete sentences and label any diagrams. Please show your work; no credit is given for solutions without work or justification.

exercise 1. *(3 points)* Use the power series of $\tan^{-1}(x)$ to prove the following expression for π as the sum of an infinite series:

$$\pi = 2\sqrt{3} \cdot \sum_{n=0}^{\infty} \frac{(-1)^n}{3^n \cdot (2n+1)}.$$

Hint: What is $\tan(\frac{\pi}{6})$?

exercise 2. *(5 points)* Use Taylor series to evaluate the limit

$$\lim_{x \to 0} \frac{\sin(x) - x}{2x^3}.$$

exercise 3. *(6 points)* Let $f(x) = \ln(1 + \frac{1}{2}x)$.

a) Write down the Maclaurin series for $f(x)$ and indicate its radius of convergence. Don't use the ratio test, use what you know about a related series.

b) Find the Maclaurin series for $\int f(x)\, dx$.

c) Write down a series that converges to $\int_0^1 f(x)\, dx$.

d) Estimate your result in c) with the first four non-zero values of the series of $\int_0^1 f(x)\, dx$.

exercise 4. *(6 points)* Describe in words the regions of $\mathbb{R}^3$ represented by the following equation(s) or inequalities. Make a simple sketch to clarify your answer.

a) $z \geq -2$.

b) $y^2 = 4$.

c) $x = z$.

d) $x^2 + y^2 = 4$ and $z = 1$.