Math 8: Calculus in one and several variables
Winter 2019 - Homework 1

Return date: Friday 01/11/19

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keywords: *Taylor polynomials, remainder estimate, geometric series*

Instructions: Write your answers neatly and clearly on straight-edged paper, use complete sentences and label any diagrams. Please show your work; no credit is given for solutions without work or justification.

exercise 1. *(4 points)* Find the Taylor polynomial $T_3(x)$, for the function $f(x)$ at a.

a) $f(x) = x + x^3$ at $a = 1$.

b) $f(x) = e^{x^2} + 1$ at $a = 1$.

exercise 2. *(4 points)* For each of the following problems, write out enough terms of the 100th Taylor polynomial

$$T_{100}(x) = c_0 + c_1 x + c_2 x^2 + \cdots + c_{100} x^{100}$$

for the function $f(x)$ at the point a, to make the pattern obvious. Then write down an explicit expression for c_n. Use whatever notation is most clear. For example, if you find that $c_0, c_1, c_2, c_3, c_4, c_5 \ldots$ is given by

$$0,\ 2,\ 6,\ 12,\ 20,\ 30, ...$$

the pattern becomes more clear if you rewrite these numbers as

$$0,\ 1 \cdot 2,\ 2 \cdot 3,\ 3 \cdot 4,\ 4 \cdot 5,\ 5 \cdot 6, ...$$

Then you can see that $c_n = n(n+1)$.

a) $f(x) = e^{2x}$ at $a = 0$.

b) $f(x) = \ln(x+1)$ at $a = 0$.

Show your work.

exercise 3. *(4 points)*

a) Find the Taylor polynomial $T_3(x)$, for the function

$$f(x) = x \cdot \ln(x) \quad \text{at the point} \quad a = 1.$$

b) For the values $0.8 \le x \le 1.2$ estimate the accuracy of the approximation using the remainder estimate

$$|R_3(x)| = |f(x) - T_3(x)|$$

in Taylor's inequality (**Theorem 11.10.9** of the book). Justify your answer.

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exercise 4. *(4 points)* Suppose we use the following estimate for $3\cos(x)$:

$$3\cos(x) \simeq 3 - \frac{3}{2}x^2.$$

a) Explain why it's okay to estimate the error using either $R_2(x)$ or $R_3(x)$. (Note that we get a better estimate using $R_3(x)$.)

b) Use the boxed statement on page 1 of the Error Estimates handout to get a bound on the error in computing $3\cos(0.1)$ using the polynomial above. Show your work.

exercise 5. *(4 points)* Determine whether the geometric series is convergent or divergent. If it is convergent, find its sum:

a) $\sum_{n=0}^{\infty} \frac{5}{\pi^n}$.

b) $\sum_{n=0}^{\infty} \frac{3^{n+1}}{(-2)^n}$.