Worksheet #22

(1) Find the indicated partial derivatives.

$$f(x,y) = \arctan(\mathrm{y/x}); \quad \mathrm{f_x}(2,3) \quad \mathrm{f_y}(2,3)$$

Solution: $f_x(x,y) = -\frac{y}{x^2}\frac{1}{1+(y/x)^2}$ $f_y(x,y) = \frac{1}{x}\frac{1}{1+(y/x)^2}$
Plugging in the point $(2,3)$, we get $f_x(2,3) = -\frac{3}{4}\frac{1}{1+(3/2)^2}$ $f_y(2,3) = \frac{1}{2}\frac{1}{1+(3/2)^2}$.

(2) Find $\frac{dz}{dx}$ and $\frac{dz}{dy}$ for $x^2 - y^2 + z^2 - 2z = 4$.
Solution: By taking the derivative of $x^2 - y^2 + z^2 - 2z = 4$ with respect to x we get

$$2x + 2z\frac{dz}{dx} - 2\frac{dz}{dx} = 0.$$

Solving for $\frac{dz}{dx}$, we find

$$\frac{dz}{dx} = \frac{-2x}{2z-2}.$$

By taking the derivative of $x^2 - y^2 + z^2 - 2z = 4$ with respect to y we get

$$-2y + 2z\frac{dz}{dy} - 2\frac{dz}{dy} = 0.$$

Solving for $\frac{dz}{dy}$, we find

$$\frac{dz}{dy} = \frac{2y}{2z-2}.$$

(3) Find $\frac{dz}{dx}$ and $\frac{dz}{dy}$ for

- $z = f(x)g(y)$
  Solution:
  $$\frac{dz}{dx} = f'(x)g(y) \quad \frac{dz}{dy} = f(x)g'(y)$$
- $z = f(x/y)$
  Solution:
  $$\frac{dz}{dx} = \frac{f'(x/y)}{y} \quad \frac{dz}{dy} = \frac{-xf'(x/y)}{y^2}$$

(4) Find all second partial derivatives. $f(x,y) = \frac{xy}{x-y}$
Solution:

$$f_x(x,y) = \frac{-y^2}{(x-y)^2}$$
$$f_y(x,y) = \frac{x^2}{(x-y)^2}$$
$$f_{xx}(x,y) = \frac{2y^2}{(x-y)^3}$$
$$f_{xy}(x,y) = \frac{-2xy}{(x-y)^3}$$
$$f_{yy}(x,y) = \frac{2x^2}{(x-y)^3}$$
$$f_{yx}(x,y) = \frac{-2xy}{(x-y)^3}$$

(5) Find f_{zyx} of $f(x,y,z) = e^{xy^2z}$
Solution:

$$f_z(x,y,z) = xy^2e^{xy^2z}$$
$$f_{zy}(x,y,z) = (2x^2y^3z + 2xy)e^{xy^2z}$$
$$f_{zyx}(x,y,z) = [(2x^2y^3z + 2xy)y^2z + 4xy^3 + 2y]e^{xy^2z}$$