Worksheet #19

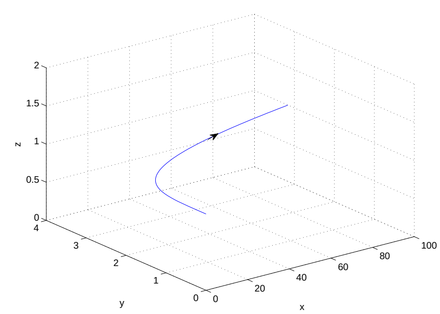
(1) Evaluate the limit.

$$\lim_{t\to 2}\left(\frac{t^2-2t}{t-2}\mathbf{i}+\sqrt{t+4}\mathbf{j}+\frac{\sin(\pi t)}{\ln(t-1)}\mathbf{k}\right)$$

Solution:

$$\lim_{t\to 2}\left(\frac{t^2-2t}{t-2}\mathbf{i}+\sqrt{t+4}\mathbf{j}+\frac{\sin(\pi t)}{\ln(t-1)}\mathbf{k}\right)=2\mathbf{i}+\sqrt{6}\mathbf{j}+\pi\mathbf{k}$$

(2) Sketch the curve $\mathbf{r}(t) =< t^2, \sqrt{t}, 1 >$. Use arrows to indicate the direction in which t increases. **Solution:**



(3) Find the unit tanget vector $\mathbf{T}(t)$ of $\mathbf{r}(t) =< \cos(t), -\sin(t), \sin(2t) >$ when $t = \pi/2$. **Solution:**

$$\mathbf{r}'(t) =< -\sin t, -\cos t, 2\cos(2t) >$$
$$\mathbf{r}'(\pi/2) =< -1, 0, -2 >$$
$$|\mathbf{r}'(\pi/2)| = \sqrt{5}$$
$$\mathbf{T}(\pi/2) = \frac{1}{\sqrt{5}} < -1, 0, -2 >$$

(4) Evaluate the integral.

$$\int_0^{\pi/2}(3\sin^2 t\cos t\mathbf{i}+2\sin t\cos^2 t\mathbf{j}+2\sin t\cos t\mathbf{k})dt$$

Solution: Breaking the integral into its 3 components, we find:

$$\int_0^{\pi/2} 3\sin^2 t\cos t dt = \sin^3 t|_0^{\pi/2} = 1-0=1$$
$$\int_0^{\pi/2} 2\sin t\cos^2 t dt = \frac{-2}{3}\cos^3 t|_0^{\pi/2} = 0-\frac{-2}{3}=\frac{2}{3}$$
$$\int_0^{\pi/2} 2\sin t\cos t dt = \int_0^{\pi/2}\sin 2t dt = \frac{1}{2}\cos 2t|_0^{\pi/2} = \frac{1}{2}-\frac{-1}{2}=1$$

Thus the integral is $< 1, \frac{2}{3}, 1 >$.