Worksheet #17

(1) Let $\mathbf{a} = -3\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$, $\mathbf{b} = -\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$, and $\mathbf{c} = 7\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$

- $\mathbf{a} \times \mathbf{b}$
  Solution:

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & 2 & -2 \\ -1 & 2 & -4 \end{vmatrix} = < -4, -10, -4 >$$

- $\mathbf{a} \times (\mathbf{b} + \mathbf{c})$
  Solution:

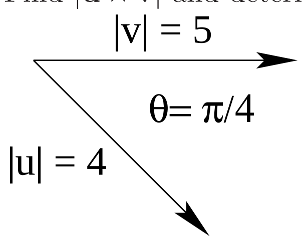
$$\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times < 6, 5, -8 > = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & 2 & -2 \\ 6 & 5 & -8 \end{vmatrix} = < -6, -36, -27 >$$

- $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c})$

Solution:

$$\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot < 6, 5, -8 > = 8$$

(2) Find $|\mathbf{u} \times \mathbf{v}|$ and determine whether $\mathbf{u} \times \mathbf{v}$ is directed into the page or out of the page.



Solution:

$$|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}||\mathbf{v}| \sin\theta = 4(5)\sin(\pi/4) = 10\sqrt{2}$$

The vector $\mathbf{u} \times \mathbf{v}$ points out of the page.

(3) Let $P(-1, 3, 1)$, $Q(0, 5, 2)$, and $R(4, 3, -1)$. Find a nonzero vector orthogonal to the plane through the points P, Q, and R.
Solution:
$\vec{PQ} = < 1, 2, 1 >$ and $\vec{PR} = < 5, 0, -2 >$.

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 1 \\ 5 & 0 & -2 \end{vmatrix} = -4\mathbf{i} + 7\mathbf{j} - 10\mathbf{k}$$

(4) Let $P(-1, 3, 1)$, $Q(0, 5, 2)$, and $R(4, 3, -1)$. Find the area of the triangle PQR.
Solution:
$A(PQR) = 1/2|\vec{PQ} \times \vec{PR}| = 1/2\sqrt{16 + 49 + 100} = \frac{\sqrt{165}}{2}$.

(5) Use the scalar triple product to determine whether the points $A(1, 3, 2)$, $B(3, -1, 6)$, $C(5, 2, 0)$, and $D(3, 6, -4)$ lie in the same plane.
Solution:
$\vec{AB} = < 2, -4, 4 >$, $\vec{AC} = < 4, -1, -2 >$, and $\vec{AD} = < 2, 3, -6 >$. The volume is given by the triple product.

$$\vec{AB} \cdot (\vec{AC} \times \vec{AD}) = \begin{vmatrix} 2 & -4 & 4 \\ 4 & -1 & -2 \\ 2 & 3 & -6 \end{vmatrix} = 2(12) - 4(20) + 4(14) > 0$$

Thus the points are not coplanar.