Worksheet #16

(1) Let $\mathbf{a} = -2\mathbf{i} + 3\mathbf{j}$ and $\mathbf{b} = 2\mathbf{i} - 3\mathbf{j}$ and $\mathbf{c} = -5\mathbf{j}$. Find the following: (a) $2\mathbf{a} - 4\mathbf{b}$ (b) $\mathbf{a} \cdot \mathbf{b}$ (c) $|\mathbf{a}|\mathbf{c} \cdot \mathbf{a}$
Solution:
(a) $2\mathbf{a} - 4\mathbf{b} =< -12, 18 >$
(b) $\mathbf{a} \cdot \mathbf{b} = -2(2) - 3(3) = -13$
(c) $|\mathbf{a}| = \sqrt{4+9} = \sqrt{13}$
$\mathbf{c} \cdot \mathbf{a} = -2(0) - 15 = -15$
$|\mathbf{a}|\mathbf{c} \cdot \mathbf{a} = -15\sqrt{13}$

(2) Find the cosine of the angle between $\mathbf{a}$ and $\mathbf{b}$ and make a sketch.
(a) $\mathbf{a} =< -1, 2 >$ $\mathbf{b} =< 6, 0 >$ (b) $\mathbf{a} =< 4, -7 >$ $\mathbf{b} =< -8, 10 >$
Solution:
(a) $\mathbf{a} \cdot \mathbf{b} = -6 \quad |\mathbf{a}| = \sqrt{5} \quad |\mathbf{b}| = 6$
$\cos\theta = \frac{-1}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$

(b) $\mathbf{a} \cdot \mathbf{b} = -32 - 70 = -102 \quad |\mathbf{a}| = \sqrt{16+49} = \sqrt{65} \quad |\mathbf{b}| = \sqrt{64+100} = \sqrt{164}$
$\cos\theta = \frac{-102}{\sqrt{65(164)}}$

(3) Write the vector $\vec{AB}$ in the form $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j}$ (a) $A(2,2), B(-3,4)$ (b) $A(0,4), B(-6,0)$
Solution:
(a) $\mathbf{a} =< -3 - 2, 4 - 2 >= -5\mathbf{i} + 2\mathbf{j}$
(b) $\mathbf{a} =< -6 - 0, 0 - 4 >= -6\mathbf{i} - 4\mathbf{j}$

(4) Show that the vectors $< 6, 3 >$ and $< -1, 2 >$ are perpendicular.
Solution:

$$\cos\theta = \frac{< 6,3 > \cdot < -1, 2 >}{| < 6, 3 > || < -1, 2 > |} = \frac{-6+6}{\sqrt{(36+9)(5)}} = 0$$

The angle between the two vectors is $\frac{\pi}{2}$. Thus they are perpendicular.

(5) Find the scalar and vector projections of $\mathbf{b}$ onto $\mathbf{a}$ where $\mathbf{a} =< 1, 1, 1 >$ and $\mathbf{b} =< 1, -1, 1 >$. Also, find the orthogonal projection.
Solution:

$$\text{comp}_{\mathbf{a}}\mathbf{b} = \frac{\mathbf{b} \cdot \mathbf{a}}{|\mathbf{a}|} = \frac{1-1+1}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\text{proj}_{\mathbf{a}}\mathbf{b} = \frac{\mathbf{b} \cdot \mathbf{a}}{|\mathbf{a}|^2}\mathbf{a} = \frac{1}{3} < 1, 1, 1 >$$

$$\text{orth}_{\mathbf{a}}\mathbf{b} = \mathbf{b} - \frac{\mathbf{b} \cdot \mathbf{a}}{|\mathbf{a}|^2}\mathbf{a} =< 1, -1, 1 > -\frac{1}{3} < 1, 1, 1 >=< 2/3, -4/3, 2/3 >$$