

### Worksheet #15

- (1) For  $A(4, 4, -1)$  and  $B(4, 1, 4)$ , find a vector  $\mathbf{a}$  with representation given by the directed line segment  $\overrightarrow{AB}$ . Draw  $\overrightarrow{AB}$  and the equivalent representation starting at the origin.

**Solution:**

$$\overrightarrow{AB} = \langle 0, -3, 5 \rangle$$

- (2) Find  $\mathbf{a} + \mathbf{b}$ ,  $2\mathbf{a} + 3\mathbf{b}$ ,  $|\mathbf{a}|$ , and  $|\mathbf{a} - \mathbf{b}|$  where  $\mathbf{a} = 2\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$ , and  $\mathbf{b} = 2\mathbf{j} - \mathbf{k}$ .

**Solution:**

$$\mathbf{a} + \mathbf{b} = \langle 2, -2, 3 \rangle$$

$$2\mathbf{a} + 3\mathbf{b} = \langle 4, -2, 5 \rangle$$

$$|\mathbf{a}| = \sqrt{4 + 16 + 16} = \sqrt{36} = 6$$

$$|\mathbf{a} - \mathbf{b}| = \sqrt{2^2 + (-4 - 2)^2 + (4 + 1)^2} = \sqrt{65}$$

- (3) Find a vector that has the opposite direction as  $\langle -2, 4, 2 \rangle$  but has length 6.

**Solution:**

First, we need to make a unit vector.

$$|\langle -2, 4, 2 \rangle| = \sqrt{4 + 16 + 4} = \sqrt{24} = 2\sqrt{6}$$

So the unit vector  $\mathbf{u}$  is  $\mathbf{u} = \langle -\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}} \rangle$

To get a vector in the opposite direction with length 6, we need to multiply  $\mathbf{u}$  by  $-6$ .

$$\text{Thus, the answer is } \langle \frac{6}{\sqrt{6}}, -\frac{12}{\sqrt{6}}, -\frac{6}{\sqrt{6}} \rangle$$

- (4) Find a unit vector that has the same direction as  $\langle -4, 2, 4 \rangle$ .

**Solution:**

$$|\langle -4, 2, 4 \rangle| = \sqrt{16 + 4 + 16} = 6$$

Thus the unit vector in the same direction is  $\langle -\frac{2}{3}, \frac{1}{3}, \frac{2}{3} \rangle$ .

- (5) If a child pulls a sled through the snow on a level path with a force of 50 N exerted at an angle of  $\frac{\pi}{4}$  above the horizontal, find the horizontal and vertical components of the force.

**Solution:**

$$\mathbf{F} = 50 \cos\left(\frac{\pi}{4}\right)\mathbf{i} + 50 \sin\left(\frac{\pi}{4}\right)\mathbf{j} = 25\sqrt{2}\mathbf{i} + 25\sqrt{2}\mathbf{j}$$