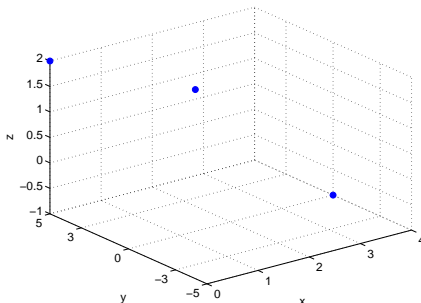


Worksheet #14

- (1) Sketch the points $(0, 5, 2)$, $(4, 0, -1)$, and $(1, -1, 2)$ on a single set of coordinate axes.

Solution:



- (2) Find the lengths of the triangle PQR . Is it a right triangle? Is it isosceles?

$$P(2, -1, 0) \quad Q(7, 0, 1) \quad R(1, 2, 1)$$

Solution:

$$\begin{aligned} |PQ| &= \sqrt{26} \\ |PR| &= \sqrt{11} \\ |QR| &= 2\sqrt{10} \end{aligned}$$

The triangle is neither right or isosceles.

- (3) Find the equation of a sphere with center $(2, -6, 4)$ and radius 5. Describe its intersection with each of the coordinate planes.

Solution:

$$(x - 2)^2 + (y + 6)^2 + (z - 4)^2 = 25$$

xy-plane $(x - 2)^2 + (y + 6)^2 = 9$

The intersection is a circle of radius 3 centered at $(2, -6, 0)$.

xz-plane $(x - 2)^2 + (z - 4)^2 = -11$

This does not make sense. Thus the sphere does not intersect the xz-plane.

yz-plane $(y + 6)^2 + (z - 4)^2 = 21$

The intersection is a circle with radius $\sqrt{21}$ centered at $(0, -6, 4)$.

- (4) Show that $x^2 + y^2 + z^2 + 8x - 6y + 2z + 17 = 0$ represents a sphere. Find its center and radius.

Solution:

$$x^2 + 8x + y^2 - 6y + z^2 + 2z = -17$$

$$x^2 + 8x + 16 + y^2 - 6y + 9 + z^2 + 2z + 9 = -17 + 16 + 9 + 1$$

$$(x + 8)^2 + (y - 3)^2 + (z + 1)^2 = 9$$

Center = $(-8, 3, -1)$ Radius $R = 3$