1. (Optimization).

   (a) Find and classify all local extreme points of $f(x, y) = x^2 + x + 2y^2$ on the domain $x^2 + y^2 < 1$.

   (b) Determine the absolute maximum and minimum of $f(x, y) = x^2 + x + 2y^2$ on the domain $x^2 + y^2 \leq 1$. Be sure to indicate both the maximum and minimum values as well as the coordinates of all points at which they occur.

2. Suppose that $z = f(x, y)$, $x = uv$ and $y = u + 3v$. Assume that when $u = 2$ and $v = 1$, $\dfrac{\partial z}{\partial u} = -2$ and $\dfrac{\partial z}{\partial v} = -1$. Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$.

3. Find an equation of the plane which is perpendicular to the line $x = 2 - t$, $y = 2t$, $z = 3 + t/2$, and which contains the line $x = 4 + 2s$, $y = -1 + 3s$, $z = 2 - 8s$.

4. Consider the surface $x^2 + y^2 + z^2 = 9$. Find the point of intersection of the tangent plane to the surface at the point $(1, 2, 2)$ and the x-axis.

5. Find the maxima and minima of $f(x, y, z) = xyz$ subject to the constraint $g(x, y, z) = x^2 + 2y^2 + 3z^2 = 6$.

6. Write an equation for the tangent plane to the level surface $f(x, y, z) = ze^{xy} + xe^{yz} = 2$ at the point $(1, 0, 1)$.

7. What is the arclength of the curve $y = \ln(\cos(x))$ for x from 0 to $\pi/4$.

8. Find the absolute extrema of $f(x, y) = e^{xy} + e^x$ in the first quadrant of the xy-plane.

9. Express the antiderivative $\displaystyle\int \frac{\sin(t^2) - t^2}{t^6}\, dt$ as an infinite series.

10. (Multiple choice — No partial credit) **Circle the correct answer.**

    (a) Find the tangent plane to the surface $z = x^2y^3$ at the point $(1, 1, 1)$.

    A. $2x + 3y - z = 4$ **B.** $3x + y - z = 3$ **C.** $x + 2y + z = 4$

    D. $2x + 3y = 5$ **E.** $3x - 2y + z = 2$

    (b) Consider the level curve of $f(x, y) = x^2 - 3y^2$ which passes through the point $(3, 1)$. Along what vector should one go to remain on the same level curve?

    A. $\langle 6, -6\rangle$ **B.** $\langle -6, 6\rangle$ **C.** $\langle -6, -6\rangle$ **D.** $\langle 0, 6\rangle$

    E. $\langle 6, 0\rangle$

(c) What is the arclength of the piece of the parabola $y = x^2$ from $(0,0)$ to $(2,4)$?

A. $\int_0^4 (1+2t)\,dt$ **B**. $\int_0^4 \sqrt{1+4t^2}\,dt$ **C**. $\int_0^2 \sqrt{t^2+t^4}\,dt$

D. $\int_0^2 \sqrt{1+4t^2}\,dt$ **E**. none of the above

(d) If $f(x,y) = \int_y^x \cos(t^3)\,dt$, then $\dfrac{\partial f}{\partial y} =$

A. $\cos(y^3)$ **B**. $3y^2\cos(y^3)$ **C**. $\sin(y^3)$

D. $-3y^2\sin(y^3)$ **E**. none of the above

(e) Suppose that you are given a function $f(x,y)$ and vectors $\mathbf{u} = \langle \frac{1}{2}, \frac{-\sqrt{3}}{2} \rangle$ and $\mathbf{v} = \langle \frac{1}{2}, \frac{\sqrt{3}}{2} \rangle$. If $(D_{\mathbf{u}}f)(x_0,y_0) = 2$ and $(D_{\mathbf{v}}f)(x_0,y_0) = -1$, then $\dfrac{\partial f}{\partial x}(x_0,y_0) =$

A. $\frac{1}{2}$ **B**. 1 **C**. 2 **D**. $\sqrt{3}$ **E**. none of the above

(f) Suppose that the graph of $z = f(x,y)$ represents the surface of a mountain, and you are standing at a point (x_0,y_0,z_0) on the surface. You are told that the gradient of f at (x_0,y_0) is $\nabla f(x_0,y_0) = \langle 1,3 \rangle$. If you move in the direction of the gradient, what is your initial angle of elevation?

A. $\tan^{-1} 3$ **B**. $\cos^{-1} 3$ **C**. $\tan^{-1}\sqrt{10}$ **D**. $\cos^{-1}\sqrt{10}$

E. none of the above