NAME : ______________________________

SECTION : (circle one) 12:30–1:35 1:45–2:50

Math 8

February 23, 2010

Midterm 2

$\boxed{\text{SOLUTIONS}}$

INSTRUCTIONS: This is a closed book exam and no notes are allowed. You are not to provide or receive help from any outside source during the exam except that you may ask the instructor for clarification of a problem. You have two hours and you should attempt all problems.

- ***Print*** your name in the space provided and circle your instructor's name.
- Sign the FERPA release on the next page *only if* you wish your exam returned in lecture.
- Calculators or other computing devices are not allowed.
- Use the blank page at the end of the exam for scratch work.
- **You must show all work and give a reason (or reasons) for your answer. A CORRECT ANSWER WITH INCORRECT WORK WILL BE CONSIDERED WRONG.**

1. (15) Evaluate $\int e^{5x+7} \sin x \, dx$.

Let $I = \int e^{5x+7} \sin x \, dx$.

Integration by parts:

$u = e^{5x+7}$ $\quad v = -\cos x$

$du = 5e^{5x+7}\, dx$ $\quad dv = \sin x \, dx$

$I = uv - \int v \, du$

$= -e^{5x+7} \cos x + 5 \int e^{5x+7} \cos x \, dx.$

Integration by parts again:

$u = e^{5x+7}$ $\quad v = \sin x$

$du = 5e^{5x+7}$ $\quad dv = \cos x \, dx$

$$I = -e^{5x+7} \cos x + 5\left(e^{5x+7} \sin x - 5 \underbrace{\int e^{5x+7} \cos x \, dx}_{I}\right).$$

$$I = -e^{5x+7} \cos x + 5e^{5x+7} \sin x - 25I$$

$$26I = \text{''}$$

$$I = \boxed{\frac{-e^{5x+7} \cos x + 5e^{5x+7} \sin x}{26} + C.}$$

2. (15) Find an equation of the plane which contains the two lines

$$\langle 1+t, 4-5t, 3t\rangle$$

and

$$\langle 2-t, -1, 3+t\rangle.$$

Normal to plane:

$$\vec{n} = \langle 1, -5, 3\rangle \times \langle -1, 0, 1\rangle$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -5 & 3 \\ -1 & 0 & 1 \end{vmatrix}$$

$$= \vec{i}\begin{vmatrix} -5 & 3 \\ 0 & 1 \end{vmatrix} - \vec{j}\begin{vmatrix} 1 & 3 \\ -1 & 1 \end{vmatrix} + \vec{k}\begin{vmatrix} 1 & -5 \\ -1 & 0 \end{vmatrix}$$

$$= -5\vec{i} - 4\vec{j} - 5\vec{k}$$

Point on plane: $(1, 4, 0)$

Plane: $\boxed{-5(x-1) - 4(y-4) - 5z = 0.}$

3. (15) Evaluate $\int \sec^3 x \tan^5 x \, dx$.

Trying to use $u = \sec x$ so $du = \sec x \tan x \, dx$.

$\sin^2 x + \cos^2 x = 1$, so $1 + \tan^2 x = \sec^2 x$,
so $\tan^2 x = \sec^2 x - 1$.

$$I = \int \sec^3 x \tan^5 x \, dx = \int \sec^2 x \left(\tan^2 x\right)^2 \sec x \tan x \, dx$$

$$= \int \sec^2 x \left(\sec^2 x - 1\right)^2 \sec x \tan x \, dx$$

$u = \sec x$
$du = \sec x \tan x \, dx$

$$I = \int u^2 (u^2 - 1)^2 \, du = \int u^2 (u^4 - 2u^2 + 1) \, du$$

$$= \int u^6 - 2u^4 + u^2 = \frac{u^7}{7} - 2\frac{u^5}{5} + \frac{u^3}{3} + C.$$

$$= \boxed{\frac{\sec^7 x}{7} - 2\frac{\sec^5 x}{5} + \frac{\sec^3 x}{3} + C.}$$

4. (15) Find an equation for the line in which the two planes

$$x + 2y + z = 5$$

and

$$2x + y - z = 7$$

intersect.

Direction of line is $\perp$ to both normals.

$$\langle 1,2,1\rangle \times \langle 2,1,-1\rangle = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 1 \\ 2 & 1 & -1 \end{vmatrix}$$

$$= \vec{i}\begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} - \vec{j}\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} + \vec{k}\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix}$$

$$= -3\vec{i} + 3\vec{j} - 3\vec{k}.$$

Point on line: try $x = 0$, then $\begin{cases} 2y + z = 5 \\ y - z = 7 \end{cases}$

so $z = 5 - 2y$, so $y - (5-2y) = 7$,

so $3y = 12$, so $y = 4$, and $z = -3$.

$(0,4,-3)$

Line: $\boxed{\langle -3,3,-3\rangle t + \langle 0,4,-3\rangle.}$

5. (15) Evaluate $\displaystyle\int \frac{1}{(1+x^2)^2}\,dx$.

From #3, $1+\tan^2 x = \sec^2 x$, so set

$$x = \tan\theta$$
$$dx = \sec^2\theta\, d\theta$$

$$\int \frac{1}{(1+x^2)^2}\,dx = \int \frac{\sec^2\theta\, d\theta}{(1+\tan^2\theta)^2}$$

$$= \int \frac{\sec^2\theta\, d\theta}{\sec^4\theta}$$

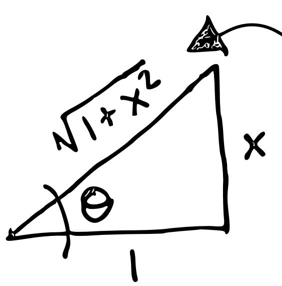
$$= \int \frac{1}{\sec^2\theta}\, d\theta$$

$$= \int \cos^2\theta\, d\theta$$

$$= \int \frac{1+\cos 2\theta}{2}\, d\theta$$

$$= \frac{\theta}{2} + \frac{\sin 2\theta}{4} + C$$

$$= \boxed{\frac{\arctan x}{2} + \frac{x}{2(1+x^2)} + C.}$$



$\sin 2\theta = 2\sin\theta\cos\theta$

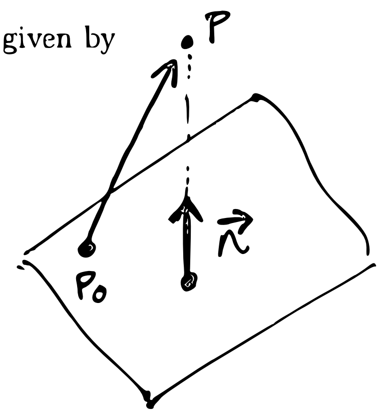
$\sin\theta = \dfrac{x}{\sqrt{1+x^2}}$

$\cos\theta = \dfrac{1}{\sqrt{1+x^2}}$

$\sin 2\theta = \dfrac{2x}{1+x^2}$

6. (15) Compute the distance from the point $(3, 4, 5)$ to the plane given by

$$2x + 3y - z = 10.$$



Normal: $\vec{n} = \langle 2, 3, -1 \rangle$.

(Any) point on plane $P_0 = (5, 0, 0)$

$\overrightarrow{P_0P} = \langle -2, 4, 5 \rangle$.

$$\text{Distance} = \left| \text{comp}_{\vec{n}} \overrightarrow{P_0P} \right|$$

$$= \left| \text{comp}_{\langle 2,3,-1 \rangle} \langle -2, 4, 5 \rangle \right|$$

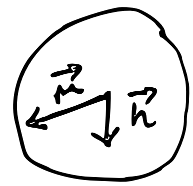
$$= \left| \frac{\langle 2, 3, -1 \rangle \cdot \langle -2, 4, 5 \rangle}{|\langle 2, 3, -1 \rangle|} \right|$$

$$= \left| \frac{-4 + 12 - 5}{\sqrt{4 + 9 + 1}} \right|$$

$$= \boxed{\frac{3}{\sqrt{14}}}$$

7. (5) Suppose that you are facing an analog clock that is showing the time 6:40. If $\mathbf{h}$ denotes the vector given by the hour hand and $\mathbf{m}$ denotes the vector given by the minute hand, does the vector $\mathbf{h} \times \mathbf{m}$ point toward you or away from you? Why?

By the right-hand rule, $\vec{h} \times \vec{m}$ points $\boxed{\text{away}}$ from you.



8. (5) Find a vector perpendicular to $\langle 1, 4, -2\rangle$. (On this problem *only*, you need not show work.)

Need $\langle 1,4,-2\rangle \cdot \langle a,b,c\rangle = 0$,

$$a + 4b - 2c = 0.$$

Setting $a=0$ and $b=1$ gives

$$\boxed{\langle 0,1,2\rangle}$$