NAME : ______________________________

SECTION : (circle one) 12:30–1:35 1:45–2:50

Math 8

February 2, 2010

Midterm 1

INSTRUCTIONS: This is a closed book exam and no notes are allowed. You are not to provide or receive help from any outside source during the exam except that you may ask the instructor for clarification of a problem. You have two hours and you should attempt all problems.

- ***Print*** your name in the space provided and circle your instructor's name.
- Mark your multiple choice answers on the final page of *this booklet*. The multiple choice booklet *will not* be collected.
- Sign the FERPA release on the next page *only if* you wish your exam returned in lecture.
- Calculators or other computing devices are not allowed.
- Use the blank page at the end of the exam for scratch work.
- **Except in the multiple choice section, you must show all work and give a reason (or reasons) for your answer. A CORRECT ANSWER WITH INCORRECT WORK WILL BE CONSIDERED WRONG.**

1. (10) Compute the Taylor polynomial of degree 4 centered at $x = 0$ for the function $f(x) = \cos 2x$.

(b) Using the Remainder Theorem, for what values of x is this Taylor polynomial guaranteed to be within $1/120$ of the true value of $f(x)$?

2. (10) What is the sum of the series $2 - 1 + \frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \cdots$?

(b) Could you make this series converge to a different sum by rearranging it? Why or why not?

3. (10) Suppose that $p > 1$. Does the series

$$\sum_{n=3}^{\infty} \frac{1}{n(\ln n)^p}$$

converge absolutely, converge conditionally, or diverge? You should mention any tests you apply, and make sure that the series satisfies the conditions of those tests.

4. (10) Find the interval of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{(x-2)^n}{6^n \sqrt{n}}.$$

5. (10) Derive a power series centered at $x = 0$ for the function

$$f(x) = \ln(1 + x).$$

(b) Find a power series centered at $x = 0$ for the function

$$f(x) = 2x^3 \ln(1 + 2x^2).$$

(You may use your answer above.)

6. (10) Express $\displaystyle\int \sin x^2 \, dx$ as a power series centered at $x = 0$.

7. (4) Suppose you know that $\lim_{n\to\infty} a_n = 0$. What can you conclude about $\sum_{n=1}^{\infty} a_n$?

A. it diverges

B. is converges

C. if the terms are positive, it converges

D. if the terms are positive and decreasing, it converges

E. nothing

8. (4) Which of the following statements are true about the series $\displaystyle\sum_{n=3}^{\infty} \frac{\ln n}{n}$?

I	The series diverges by the Ratio Test.
II	The series diverges by comparison to the harmonic series.
III	The series diverges by the Test for Divergence.

A. None

B. I only

C. II only

D. III only

E. I and II only

F. I and III only

G. II and III only

H. I, II, and III

9. (4) What is the interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{(x+3)^n}{2^n n!}$?

A. $(-5, -1]$

B. $[-5, 1]$

C. $[-1, 5]$

D. $[-1, 5)$

E. $(-\infty, \infty)$

10. (4) Which of the following series converge (either absolutely or conditionally)?

I	II	III
$\sum_{n=1}^{\infty}(-1)^n \frac{\sqrt{n^3+2}}{8n+11}$	$\sum_{n=1}^{\infty} \frac{\sin^6 n}{\pi^n}$	$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)(\ln\ln n)}$

A. None

B. I only

C. II only

D. III only

E. I and II only

F. I and III only

G. II and III only

H. I, II, and III

11. (4) Suppose that the series

$$f(x) = 3 + 2(x-1) + \frac{(x-1)^2}{7} - \frac{(x-1)^3}{3} + \frac{(x-1)^4}{12} + \cdots$$

converges for all values of x. What is $f'''(1)$?

A. 3

B. $1/3$

C. $-1/3$

D. 2

E. -2

12. (4) Which of the following series converge *absolutely*?

I	II	III
$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{3/2}}$	$\sum_{n=1}^{\infty} \left(\frac{-5}{6}\right)^n$	$\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$

A. None

B. I only

C. II only

D. III only

E. I and II only

F. I and III only

G. II and III only

H. I, II, and III

13. (4) What is the Taylor series centered at $x = 0$ for the function

$$f(x) = -xe^{-x^2}?$$

A. $\displaystyle\sum_{n=0}^{\infty}(-1)^{n+1}\frac{x^{2n+1}}{n!}$

B. $\displaystyle\sum_{n=0}^{\infty}(-1)^{n+1}\frac{x^{2n+1}}{(2n+1)!}$

C. $\displaystyle\sum_{n=0}^{\infty}(-1)^{n}\frac{x^{2n+1}}{(2n+1)!}$

D. $\displaystyle\sum_{n=0}^{\infty}(-1)^{n}\frac{x^{2n}}{(2n)!}$

E. $\displaystyle\sum_{n=0}^{\infty}(-1)^{n}\frac{x^{2n}}{n!}$

14. (4) Which of the following statements are true about the sequence $\{1/n\}$?

I	The sequence converges.
II	The sequence is monotone.
III	The sequence is bounded.

A. None

B. I only

C. II only

D. III only

E. I and II only

F. I and III only

G. II and III only

H. I, II, and III

15. (4) Two 50% marksmen decide to fight in a duel in which they exchange shots until one of them is hit. What is the chance that the first shooter wins?

A. $1/3$

B. $1/2$

C. $2/3$

D. $3/4$

E. 1

16. (4) Suppose that $\sum_{n=0}^{\infty} c_n 4^n$ converges and $\sum_{n=0}^{\infty} c_n(-5)^n$ diverges. Which of the following statements is correct?

I	The series $\sum_{n=0}^{\infty} c_n 3^n$ converges.
II	The series $\sum_{n=0}^{\infty} c_n(-2)^n$ converges.
III	The series $\sum_{n=0}^{\infty} c_n 6^n$ diverges.

A. None

B. I only

C. II only

D. III only

E. I and II only

F. I and III only

G. II and III only

H. I, II, and III