

1. (10) Determine if the improper integral

$$\int_0^{\infty} \frac{x}{1+x^2} dx$$

converges or diverges.

2. (12) Determine if the series

$$\sum_{n=2}^{\infty} \frac{\sqrt{2n}}{n^2 - 1}$$

converges. Mention any test that you might use and verify that it is applicable.

3. (14) The following power series has radius of convergence  $R = 7$ .

$$\sum_{n=1}^{\infty} \frac{(x-2)^n}{\sqrt[3]{n} 7^n}$$

Find the interval of convergence. Mention any test that you might use and verify that it is applicable.

4. (12) Find the radius of convergence of the series

$$\sum_{n=1}^{\infty} \frac{nx^n}{(n+1)!2^{n-1}}.$$

5. (10) Suppose that  $f(x)$  is equal to its Taylor series

$$f(x) = \sum_{n=0}^{\infty} \frac{1}{2^n} (x - 3)^n$$

about  $a = 3$ . What is the 39th derivative  $f^{(39)}(3)$ ? You need not simplify your answer. No partial credit will be given for this problem.

6. (12) Write down the first three non-zero terms of the Taylor series for  $\ln(2x + 4)$  at  $a = 1$ .

7. (12) Express the integral

$$\int 2(2+x)^{-1} dx$$

as a MacLaurin series. It suffices to write down the first four non-zero terms. You may assume that the arbitrary constant  $C = 0$ .

8. (18) For each of the following statements, fill in the blank with the letters **T** or **F** depending on whether the statement is true or false. You do not need to show your work and no partial credit will be given on this problem.

(a) The sequence  $\left\{\left(\frac{\pi}{3}\right)^n\right\}$  converges.

ANS: \_\_\_\_\_

(b) The series  $\sum_{n=1}^{\infty} a_n$  converges if and only if  $\lim_{n \rightarrow \infty} (a_1 + a_2 + \cdots + a_n)$  exists.

ANS: \_\_\_\_\_

(c) The series  $.9 + .99 + .999 + \cdots$  converges to 1.

ANS: \_\_\_\_\_



(d) If  $\sum_{n=1}^{\infty} a_n$  is a divergent series, then  $\sum_{n=1}^{\infty} |a_n|$  is a divergent series.

ANS: \_\_\_\_\_

(e)  $\lim_{n \rightarrow \infty} (\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^n}) = 1$ .

ANS: \_\_\_\_\_

(f) If  $0 \leq a_n \leq b_n$  and  $\sum_{n=1}^{\infty} b_n$  converges, then  $\sum_{n=1}^{\infty} a_n$  converges.

ANS: \_\_\_\_\_

NAME : \_\_\_\_\_  
SECTION : (circle one) Arkowitz (10 hour) Weber (11 Hour) Mainkar (12 hour)

## Math 8

22 October 2007  
Hour Exam I

INSTRUCTIONS: This is a closed book exam and no notes are allowed. You are not to provide or receive help from any outside source during the exam except that you may ask the instructor for clarification of a problem. You have two hours and you should attempt all problems.

- **Except in Problems 5 and 8, you must show all work and give a reason (or reasons) for your answer. A correct answer with incorrect work will be considered wrong.**
  - *Print* your name in the space provided and circle your instructor's name.
  - Sign the FERPA release on the next page *only if* you wish your exam returned in lecture.
  - Calculators or other computing devices are not allowed.
  - Use the blank page at the end of the exam for scratch work.
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FERPA RELEASE: Because of privacy concerns, we are not allowed to return your graded exams in lecture without your permission. If you wish us to return your exam in lecture, please sign on the line indicated below. Otherwise, you will have to pick your exam up in your instructor's office after the exams have been returned in lecture.

SIGN HERE: \_\_\_\_\_.

Problem	Points	Score
1	10	
2	12	
3	14	
4	12	
5	10	
6	12	
7	12	
8	18	
Total	100	