Math 8, Winter 2005

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Numerical Methods

Last time, we talked about two new numerical methods for approximating integrals:

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Last time, we talked about two new numerical methods for approximating integrals:

- Midpoint rule:

$$\int_a^b f(x)\,dx \sim M_n = \Delta x(f(\overline{x}_1) + f(\overline{x}_2) + \ldots f(\overline{x}_n))$$

where

$$\overline{x}_i = \frac{x_{i-1} + x_i}{2}$$

With error bound:

$$E_M(n) \leq \frac{K(b-a)^3}{12n^2}$$

- Trapezoidal rule:

$$\int_a^b f(x)\,dx \sim T_n = \frac{\Delta x}{2}(f(x_0)+2f(x_1)+2f(x_2)+\cdots+2f(x_{n-1})+f(x_n))$$

With error bound:

$$E_T(n) \leq \frac{K(b-a)^3}{24n^2}$$

Simpson's Rule

Next, we introduce one last approximation technique, Simpson's rule. In the Trapezoidal rule, we approximated the curve with straight lines. In Simposoin's rule, we approximate by parabolas.

- The area under the parabola passing through $(x_i, f(x_i))$, $(x_{i+1}, f(x_{i+1}))$ and $(x_{i+2}, f(x_{i+2}))$ for $x_i \leq x \leq x_{i+2}$ is

$$\frac{\Delta x}{3}(f(x_i) + 4f(x_{i+1}) + f(x_{i+2}))$$

- Summing over all parabolae yields Simposon's rule:

$$\int_a^b f(x)\,dx \sim S_n = \frac{\Delta x}{3}(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \dots \\ + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)$$

- We also have an error bound:

$$E_S(n) \leq \frac{K(b-a)^5}{180n^4}$$

Example

- Estimate the number of terms needed to evaluate

$$\int_0^1 e^{-x^2}\, dx$$

  to within 0.01 for Midpoint, Trapezoidal and Simpson's rules.

- For the rule with fewest needed terms, estimate the integral.