Math 8, Winter 2005

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Planes

A plane is determined by a point, $\vec{r}_0$, on the plane and a vector, $\vec{n}$, perpendicular to the plane.

$$\vec{n} \cdot (\vec{r} - \vec{r}_0) = 0$$

If $\vec{n} =< a, b, c >$, $\vec{r}_0 =< x_0, y_0, z_0 >$ and $\vec{r} =< x, y, z >$ we have the scalar equation of the plane:

$$< a, b, c > \cdot (< x, y, z > - < x_0, y_0, z_0 >) = 0$$

$$< a, b, c > \cdot < x - x_0, y - y_0, z - z_0 > = 0$$

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

Examples

- Find the plane passing through the point $P = (1,1,1)$, $Q = (-2,3,1)$, and $R = (3,0,2)$.
- Find the angle between the planes $x+y+z = 1$ and $x-2y-3z = 1$.
- Find the equation of the line given as the intersection of these two planes.
- Compute the distance between the point $P = < x_1, y_1, z_1 >$ and the plane $ax + by + cz + d = 0$.

Space curves

A space curve is a vector valued function given in components as

$$\vec{r}(t) =< f(t), g(t), h(t) >$$

Examples:

- Helix:

$$< \cos(t), \sin(t), t >$$

- $$< t, t^2, t^3 >$$

- $$< t, t, \cos(t) >$$

Limits and derivatives

For a vector valued function of one variable (e.g. a space curve), we define limits and derivatives by using the usual limit and derivative operations on each coordinate. If

$$F : \mathbb{R} \to \mathbb{R}^k$$

$$t \to < f_1(t), \ldots, f_k(t) >$$

then

$$\lim_{t \to t_0} F(t) = < \lim_{t \to t_0} f_1(t), \ldots, \lim_{t \to t_0} f_k(t) >$$

and

$$\frac{d}{dt} F(t) = < \frac{d}{dt} f_1(t), \ldots, \frac{d}{dt} f_k(t) >$$

Smoothness

We call a vector valued function of one variable, F, *smooth* if F' is continuous and $F'(t) \neq 0$ for any t in the domain of F.

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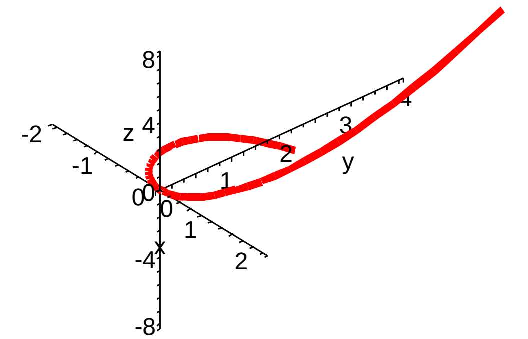

Example: Where is the curve $\vec{r}(t) =< t, t^2, t^3 >$ smooth?

Answer:

$$\vec{r}'(t) =< 1, 2t, 3t^2 >$$

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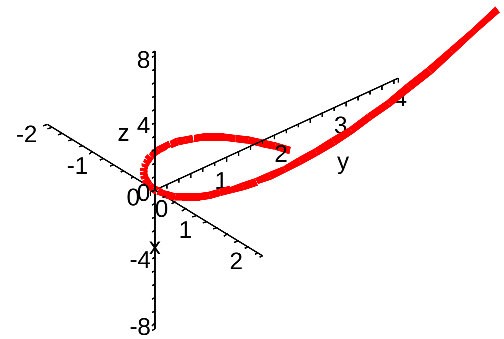
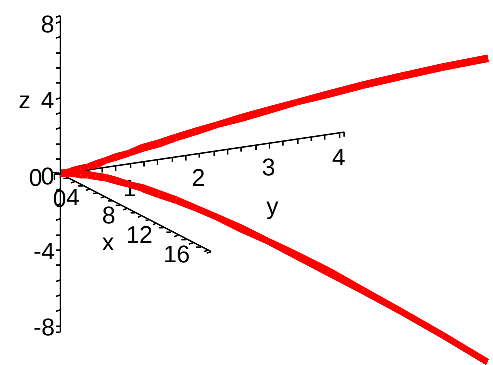


Example: Where is the curve $\vec{r}(t) =< t^4, t^2, t^3 >$ smooth?

Answer:

$$\vec{r}'(t) =< 4t^3, 2t, 3t^2 >$$

It is smooth everywhere except at $t = 0$.



Arclength

To measure the length of a curve in the plane given by $\vec{r}(t) =< f(t), g(t) >$ for $a \leq t \leq b$, we compute the following integral:

$$L = \int_a^b \sqrt{f'(t)^2 + g'(t)^2}\, dt$$

while for a spacecurve given by $\vec{r}(t) =< f(t), g(t), h(t) >$ for $a \leq t \leq b$, we compute:

$$L = \int_a^b \sqrt{f'(t)^2 + g'(t)^2 + h'(t)^2}\, dt$$

In vector notation we summarize this as:

$$L = \int_a^b |\vec{r}\,'(t)|\, dt$$

Example: compute the arclength of $\vec{r}(t) =< \cos(t), \sin(t), t^2 >$ for $0 \leq t \leq 1$