Math 8, winter 2005

Scott Pauls

Dartmouth College, Department of Mathematics

1/5/05

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Administrivia

- Instructor: Scott Pauls
- Course webapge: http://www.math.dartmouth.edu/ m8w05
- Office: 404 Bradley Hall
- Phone: 646-1047, email: scott.pauls@dartmouth.edu
- Office hours: Wednesday 3-4:30pm, Friday 9-11am

Administrivia

- This is section 2, MWF 1:45-2:50
- Classroom: Moore B03
- xhour: Thursday 1-1:50pm
- Text: Stewart's *Calculus*, fifth edition.

Course Structure

- Four components of your grade:
  - Midterm (2 hour): 20 percent
  - Two quizzes (50 minutes each): 20 percent (together)
  - Final exam: 40 percent
  - Homework: 20 percent
- Each quiz will be at a natural break in the material.
- Final exam is cumulative but may emphasize later material.
- Homework is assigned and due via webwork.
- We'll have homework sets assigned every class and due once a week.
- Webwork demonstration tomorrow in xhour, 1-1:50pm.

Important Dates

- Jan. 4 - first day of class
- Jan. 17 - MLK day, no class
- Jan. 18 - NRO, add/drop deadline
- Jan. 20 - xhour makeup for MLK day
- Jan. 27 - **Quiz 1 during xhour**
- Feb. 10 - **Midterm exam, 6-8pm**
- Feb. 14 - Withdraw deadline (without W)
- Feb. 23 - Final chance to withdraw (with W)
- Feb. 24 - **Quiz 2 during xhour**
- March 9 - Last day of class
- March 12 - **Final exam, 3-6pm**

Expectations

- Assigned reading should be completed before class.
- Having trouble with the material?
  - Come to office hours (W 3-4:30, F 9-11) (or make an appointment)
  - Go to tutorials: Sun, Tues, Thurs evenings 7-9pm. Lcoation TBA.
  - Other options: tutors, study groups, etc.
- Don't fall behind!

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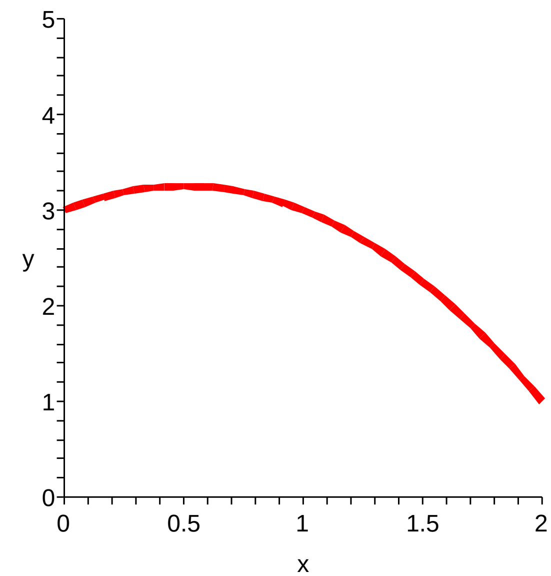
- We use Riemann sums to approximate the area:

$$\sum_{i=1}^{n} f(x_i)\Delta x$$



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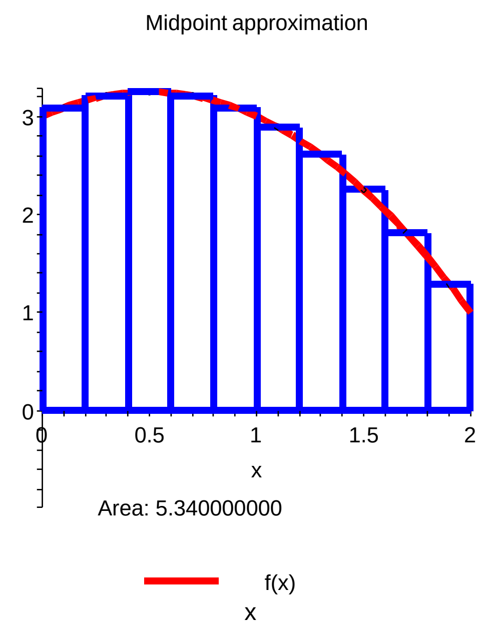

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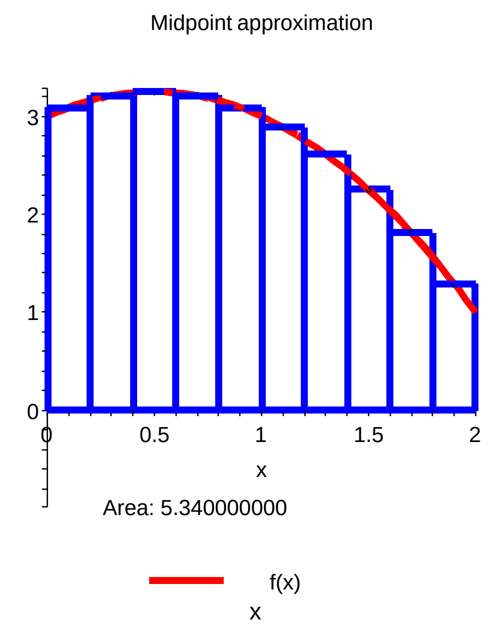

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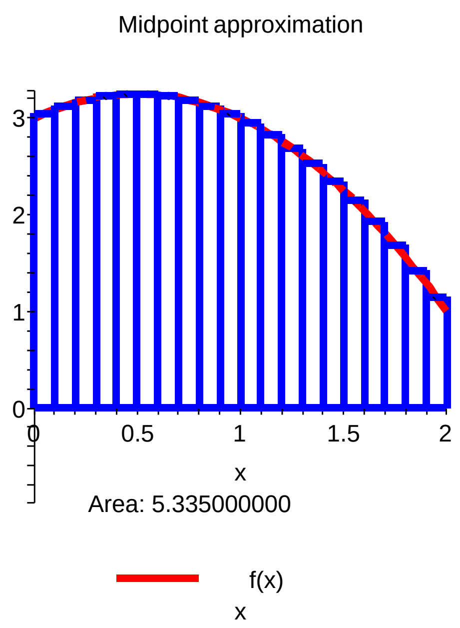

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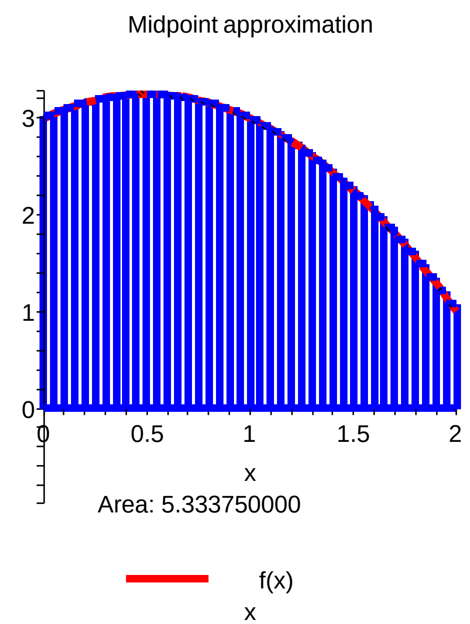

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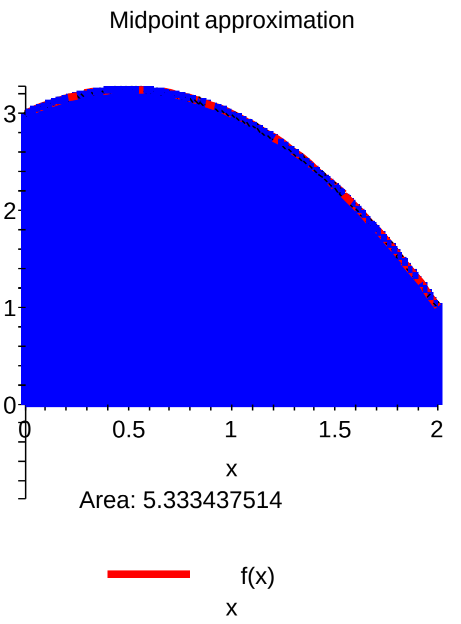

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- Several potential problems:
  - A sum with infinitely many terms (we will return to this later in the course)
  - $\Delta x \to 0$
  - What does this mean?

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- Thus, to calculate integrals easily, we'd like to find anti-derivatives of any function (yet another topic we will return to this term).

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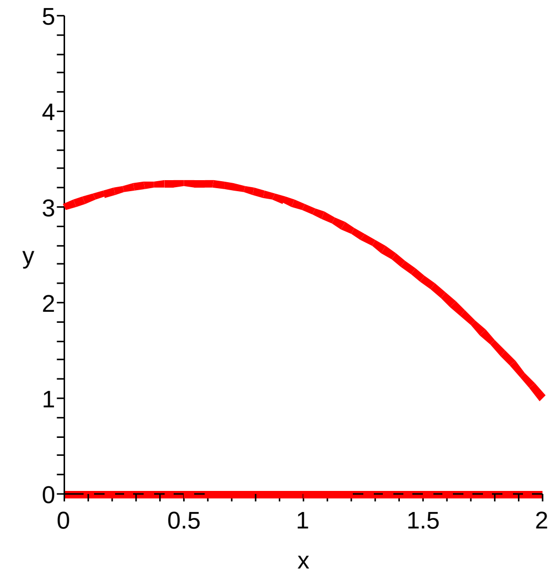

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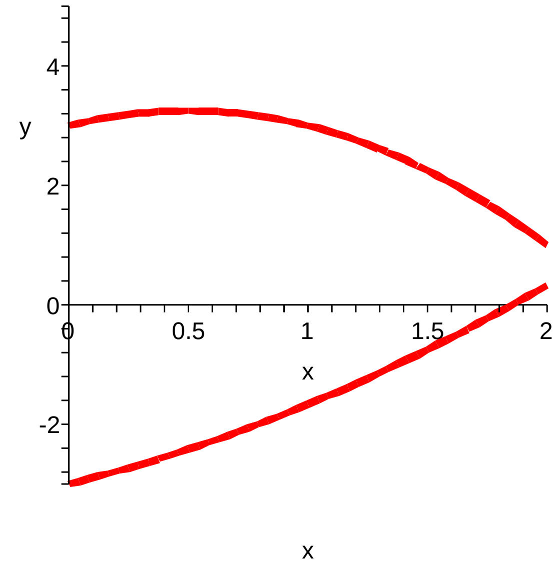

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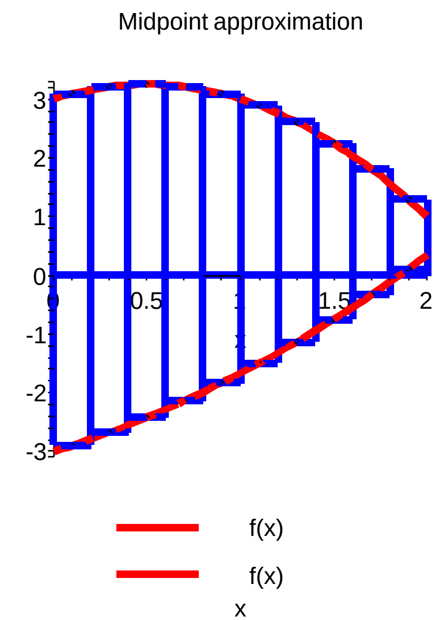

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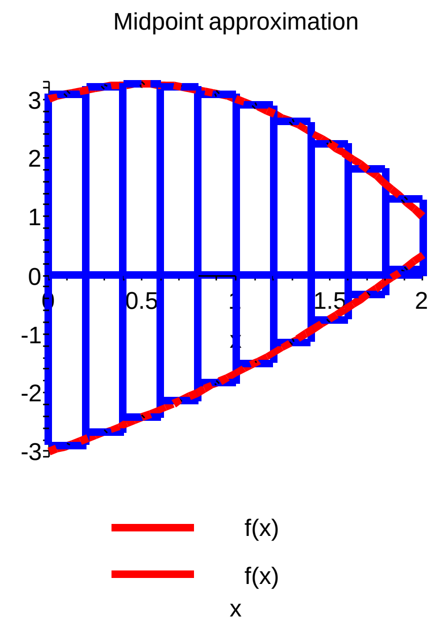
- All that changes in our Riemann sum is that the height of the box is now given by $f(x_i) - g(x_i)$.
- In other words, an approximation of the area between f and g is

$$\sum_{i=1}^{n} (f(x_i) - g(x_i))\Delta x$$



Area between two curves

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$$A = \int_a^b |f(x) - g(x)|\, dx$$

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- Find the area of the region bounded by $x = y^2$, $y = x + 5$, $y = 2$ and $y = -1$.

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- Consider the area between the curve $y = 2x - x^2$ and the x-axis. A line through the origin cuts this region into two pieces. Find the line that cuts the region into two pieces of equal area.

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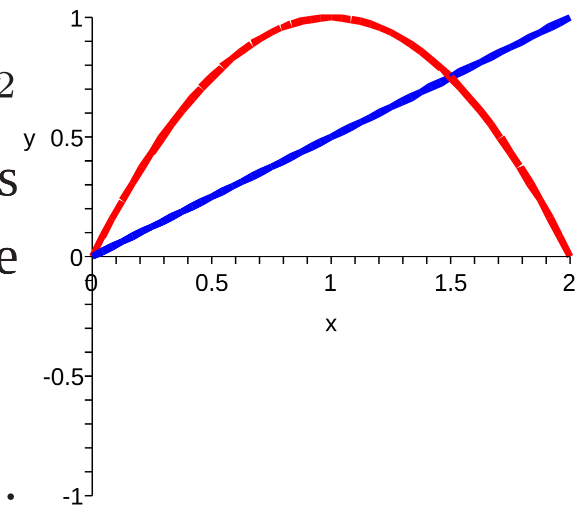

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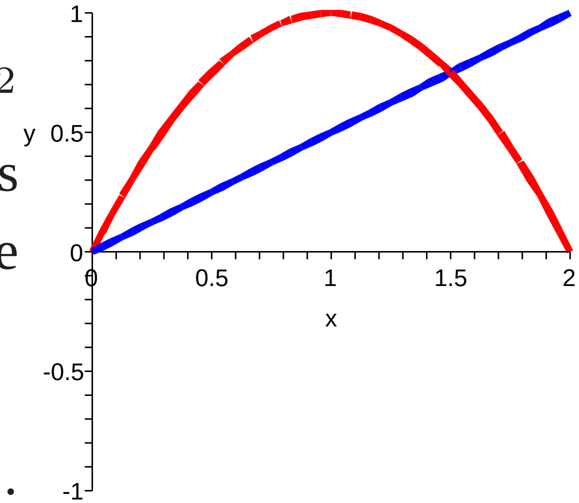

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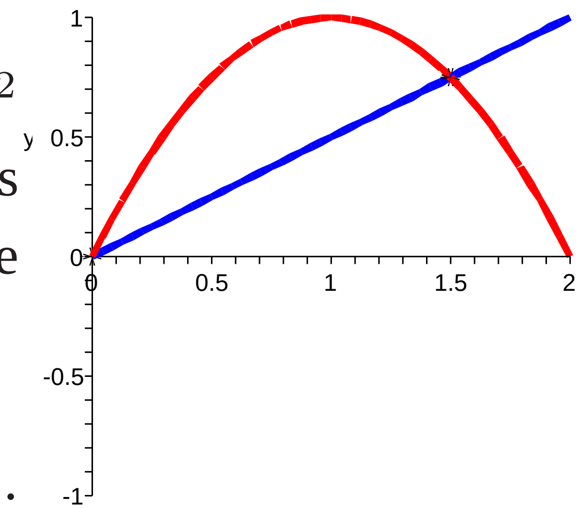

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- To find A_1 and A_2, we must find the points of intersection.

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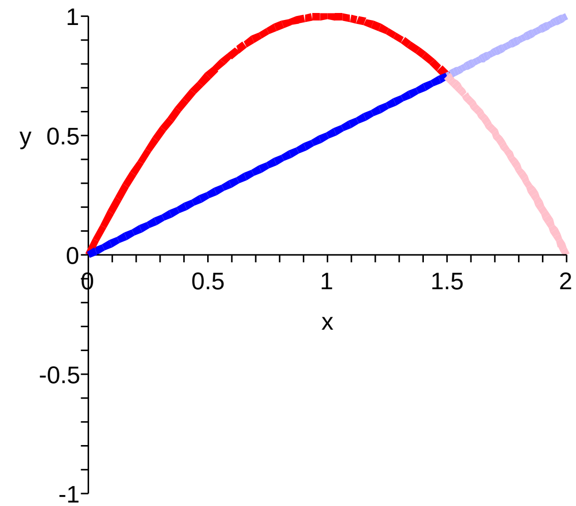

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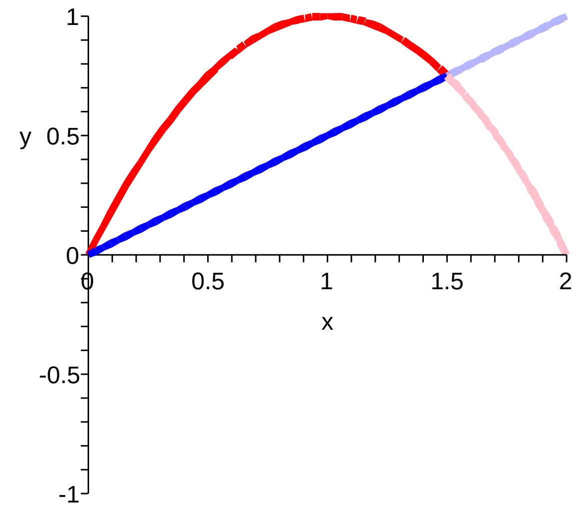

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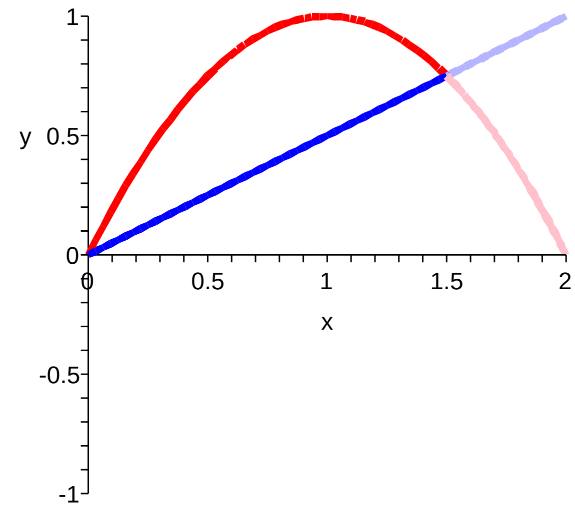

- The points occur at $x = 0$ and $x = 2 - m$.
- For R_1, we have $f(x) = 2x - x^2$, $g(x) = mx$ and $x \in [0, 2 - m]$.

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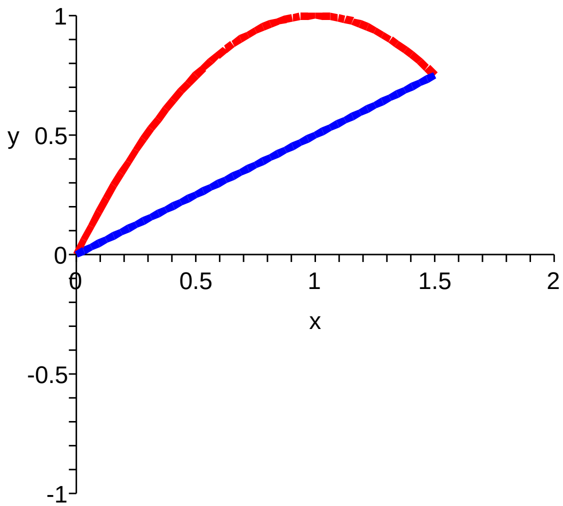
$$\int_0^{2-m} (2x - x^2) - mx\, dx = \int_0^{2-m} (2-m)x - x^2\, dx$$

$$= \left(\frac{(2-m)}{2} x^2 - \frac{x^3}{3} \right) \Big|_0^{2-m}$$

$$= \frac{(2-m)^3}{2} - \frac{(2-m)^3}{3}$$

$$= \frac{(2-m)^3}{6}$$



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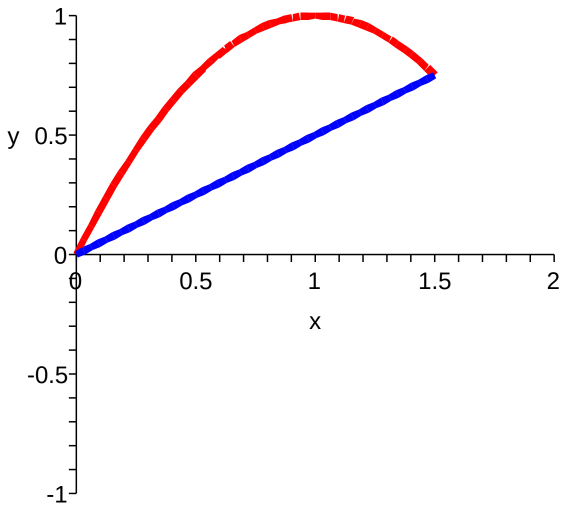
So,

$$A_1 = \frac{(2-m)^3}{6}$$



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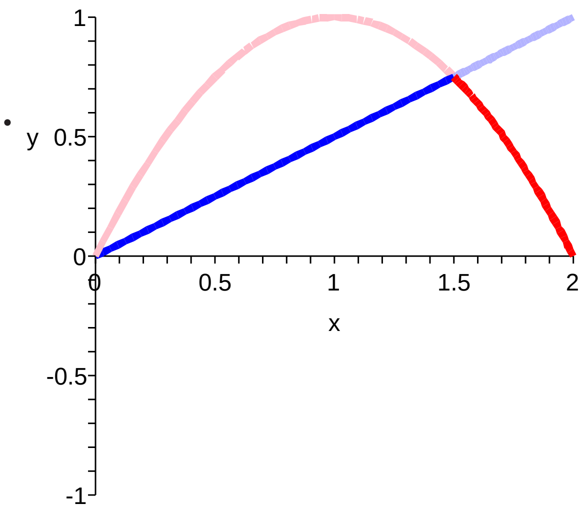

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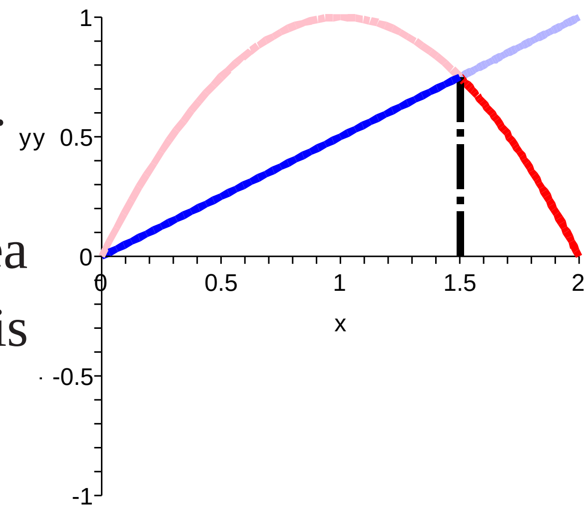

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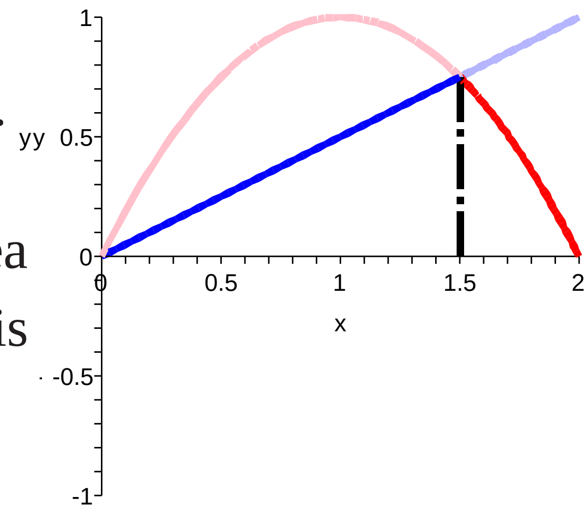

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- Now the right hand portion: it is just the area under the curve $y = 2x - x^2$ for $x \in [2-m, 2]$. The area is $\frac{4}{3} - (2-m)^2 + \frac{1}{3}(2-m)^3$

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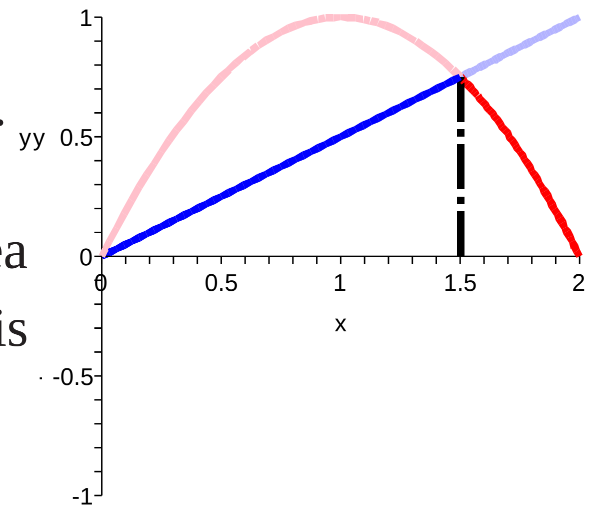

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- $$A_2 = -\frac{(2-m)^3}{6}$$

Example (cont)

Putting this together we have that

$$A_1 = \frac{(2-m)^3}{6}$$

$$A_2 = -\frac{(2-m)^3}{6}$$

For the two to be equal we must have that $2 - m = 0$ or $m = 2$.