MATH 8 PRACTICE EXAM PROBLEMS

Disclaimer: This set of problems is meant neither to indicate the length nor composition of the actual exam. However, they may give an indication of the type of problems which will appear on the exam.

1. Determine whether the following integrals converge or diverge.

$$\int_1^\infty \frac{dx}{x+e^{2x}}, \qquad \int_2^3 \frac{dx}{\sqrt{3-x}}, \qquad \int_0^\infty \frac{x\,dx}{1+x^2}, \qquad \int_0^\infty \frac{dx}{\sqrt[3]{x}+x^3}.$$

2. Compute the Taylor polynomial of degree 5 for $f(x) = \sqrt{x}$ at $a = 4$.

3. Determine whether the following sequences converge or diverge. If convergent, determine the value.

$$a_n = \frac{n\cos(n)}{1+n^2}, \qquad b_n = n\sin(1/n), \qquad c_n = (-1)^n e^{-n}.$$

4. Determine whether the following series converge absolutely, converge conditionaly or diverge.

$$\sum_{n=3}^\infty \frac{2^{2n+1}}{3^n}, \qquad \sum_{n=1}^\infty \frac{n^2+3n-2}{5n^2+7n+2}, \qquad \sum_{n=2}^\infty \frac{1}{n\ln(n)},$$

$$\sum_{n=2}^\infty \frac{\ln(n)}{n^3}, \qquad \sum_{n=2}^\infty \frac{(-1)^n}{\ln(n)}, \qquad \sum_{n=1}^\infty \frac{3^n n!}{1\cdot 3\cdot 5\cdots(2n-1)}.$$

5. Determine the radius of convergence and the interval of convergence of the following series.

$$\sum_{n=0}^\infty \frac{(2-x)^n}{2^{n+1}}, \qquad \sum_{n=1}^\infty \frac{2^n(x-1)^n}{n!}, \qquad \sum_{n=2}^\infty \frac{\sqrt{n+1}}{n-1}(x+2)^n.$$

6. Find the MacLaurin series for the function $y = x\sin(x^2)$. Find the Taylor series for the function $y = (x-3)^2 e^x$ about $a = 3$. (It suffices to write down the first four terms of each series.)

7. Consider the problem of approximating $\int_a^b f(x)dx$ using the midpoint rule with n subintervals. Recall that we may give an upper bound for error E_n in this approximation:

$$E_n \leq \frac{M_2(b-a)(\Delta x)^2}{24},$$

where $\Delta x = \dfrac{b-a}{n}$ and M_2 is a nonnegative constant such that $|f''(x)| \leq M_2$ for every x in $[a,b]$. How many subintervals are required with the midpoint rule to give the value of

$$\int_0^3 e^{x^2}\,dx$$

with an error less that 10^{-4}?

8. Indicate whether each of the following statements is TRUE or FALSE.

___(a) If the series $\sum_{n=1}^{\infty} a_n$ is convergent, then $\lim_{n\to\infty} a_n = 0$.

___(b) If $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are both divergent series, then $\sum_{n=1}^{\infty}(a_n + b_n)$ is divergent.

___(c) If series $\sum_{n=1000}^{\infty} a_n$ is convergent, then the series $\sum_{n=1}^{\infty} a_n$ is also convergent.

___(d) If the sequence $\{a_n\}$ is convergent, then the sequence $\{a_n\}$ is bounded.

___(e) If $\{a_n\}$ is an increasing sequence, then the sequence $\{a_n\}$ is convergent.

___(f) If $a_n \geq 0$ for every $n \geq 1$, and the sequence $\{a_n\}$ is decreasing, then $\{a_n\}$ is convergent.

___(g) If $\lim_{n\to\infty} a_n \neq 0$, then the sequence $\{a_n\}$ is divergent.

___(h) If $f(x) \geq g(x) \geq 0$ for every $x \geq a$, and $\int_a^{\infty} f(x)dx$ is convergent, then $\int_a^{\infty} g(x)dx$ is convergent.

___(i) If $\lim_{t\to\infty} \int_{-t}^{t} f(x)dx = 0$, then $\int_{-\infty}^{\infty} f(x)dx = 0$.