PROPERTIES OF VECTOR OPERATIONS HANDOUT

APRIL 15, 2019

Theorem. Let $\vec{u}, \vec{v}$ and $\vec{w}$ be vectors and r and s be scalars.

(i) (Commutativity): $\vec{u} + \vec{v} = \vec{v} + \vec{u}$
(ii) (Associativity): $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$
(iii) (Additive identity): $\vec{u} + \vec{0} = \vec{u}$
(iv) (Additive inverse): $\vec{u} + (-\vec{u}) = \vec{0}$
(v) (Associativity of scalar multiplication): $r(s\vec{u}) = (rs)\vec{u}$.
(vi) (Distributivity): $(r + s)\vec{u} = r\vec{u} + s\vec{u}$
(vii) (Distributivity): $r(\vec{u} + \vec{v}) = r\vec{u} + r\vec{v}$
(viii) (Multiplicative identity): $1\,\vec{u} = \vec{u}$
(ix) (Zero property): $0\,\vec{u} = \vec{0}$

Proposition 1.

1. The plane in $\mathbb{R}^3$ that is parallel to the xy-plane and contains the point (a, b, c) has equation $z = c$.
2. The plane in $\mathbb{R}^3$ that is parallel to the xz-plane and contains the point (a, b, c) has equation _____.
3. The plane in $\mathbb{R}^3$ that is parallel to the yz-plane and contains the point (a, b, c) has equation _____.

1. Let $\vec{v} = \langle 2, -6 \rangle$ and $\vec{w} = \langle 1, 4 \rangle$.
   (a) Calculuate the component form for $4\vec{v} - 3\vec{w}$.

   (b) What is the length of $4\vec{v} - 3\vec{w}$?

   (c) Find a unit vector in the same direction as $4\vec{v} - 3\vec{w}$.

2. An airplane is flying east at 400mph when the wind begins blowing 30 degrees south of east at a speed of 50mph.
   (a) After the wind begins blowing, what is the new speed and direction of the airplane?

   (b) What direction should the pilot fly in order to keep the course due east?

3. A sphere contains the points $P_1 = (4, -2, 0)$ and $P_2 = (1, 1, 8)$.
   (a) What is the center of the sphere?

   (b) What is the radius of the sphere?

   (c) What is an equation for the sphere?

4. Consider the equation $(x+3)^2 + (y-1)^2 = 9$.
   (a) What shape does this equation define in $\mathbb{R}^2$?

   (b) What surface does this equation define in $\mathbb{R}^3$?