DIRECTIONAL DERIVATIVES AND THE GRADIENT HANDOUT

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Theorem. Suppose that $f(x,y)$ is differentiable and $\vec{u}$ is a unit vector. Then

$$D_{\vec{u}}f(x,y) = \nabla f(x,y) \cdot \vec{u}\,.$$

Exercise 1. Let $f(x,y) = x^2y^3 - 4y$ and $\vec{v} = \langle 2,5\rangle$. Find the directional derivative of f at the point $(2,-1)$ in the direction of $\vec{v}$.

Exercise 2. Let $f(x,y) = x\sin(xy)$. Find the directional derivative of f at the point $(2,0)$ in the direction of $\theta = \pi/3$ (as measured counterclockwise from the x-axis).

Exercise 3. Let $g(x,y) = x^2 + y^2 - 4x$.
(a) What is the equation of the level curve passing through the point $(1,2)$?
(b) Find a tangent vector to this level curve.

Exercise 4. Let $f(x,y,z) = xy + yz + zx$. Compute the tangent plane to the level surface $f(x,y,z) = 3$ at the point $(1,1,1)$.

Exercise 5. The normal line of a surface at a point is the line that is perpendicular to the surface at that point. Compute the equations of the tangent plane and normal line at the point $(-2,1,-3)$ to the ellipsoid

$$\frac{x^2}{4} + y^2 + \frac{z^2}{9} = 3\,.$$