LIMITS AND CONTINUITY HANDOUT

MAY 3, 2019

Theorem (Limit laws)**.** Let $f(x,y)$ and $g(x,y)$ be defined in a neighborhood around (a,b) and let c be a constant. Assume that $\lim_{(x,y)\to(a,b)} f(x,y) = L$ and $\lim_{(x,y)\to(a,b)} g(x,y) = M$. Then

(i) $\lim_{(x,y)\to(a,b)} c = c$ (constant law)

(ii) $\lim_{(x,y)\to(a,b)} x = a$ and $\lim_{(x,y)\to(a,b)} y = b$

(iii) $\lim_{(x,y)\to(a,b)} (f(x,y) \pm g(x,y)) = L \pm M$ (sum and difference laws)

(iv) $\lim_{(x,y)\to(a,b)} f(x,y)g(x,y) = LM$ (product law)

(v) $\lim_{(x,y)\to(a,b)} \dfrac{f(x,y)}{g(x,y)} = \dfrac{L}{M}$, provided $M \neq 0$ (quotient law)

(vi) $\lim_{(x,y)\to(a,b)} \sqrt[n]{f(x,y)} = \sqrt[n]{L}$ for all L if n is odd and positive, and for $L \geq 0$ if n is even and positive (root law)

Exercise 1. For each of the below, either find the limit if it exists, or show that it does not exist.

(a) $\lim_{(x,y)\to(0,0)} \dfrac{6x^3y}{2x^4+y^4}$

(b) $\lim_{(x,y)\to(2,1)} \dfrac{x-y-1}{\sqrt{x-y}-1}$

(c) $\lim_{(x,y)\to(0,0)} \dfrac{x^3-y^3}{x-y}$

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Exercise 2. Let $f(x,y) = x^2 - 4y$.

(a) Compute $\lim\limits_{h \to 0} \dfrac{f(x+h,y) - f(x,y)}{h}$.

(b) Compute $\lim\limits_{h \to 0} \dfrac{f(x,y+h) - f(x,y)}{h}$.