ARC LENGTH AND FUNCTIONS OF SEVERAL VARIABLES HANDOUT

MAY 1, 2019

Theorem (Arc length)**.** Given a smooth curve C parametrized by a function $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$ with $a \leq t \leq b$, the arc length of C over the interval $[a, b]$ is

$$s = \int_a^b \sqrt{(f'(t))^2 + (g'(t))^2 + (h'(t))^2}\, dt = \int_a^b \|\vec{r}\,'(t)\|\, dt\,.$$

Definition 1. Given a function $f(x, y)$ and a number c in the range of f, a *level curve* of f for the value c is the set of points (x, y) satisfying the equation $f(x, y) = c$.

Definition 2. A *vertical trace* of a function $f(x, y)$ is the set of points such that the equation $f(a, y) = z$ for some constant a, or $f(x, b) = z$ for some constant b.

Exercise 1. Let C be the helix defined by the function $\vec{r}(t) = \langle 3\cos(t), 3\sin(t), t \rangle$. Find the length of the segment of the curve between $(3, 0, 0)$ and $(3, 0, 2\pi)$.

Exercise 2. Let $g(x, y) = \sqrt{16 - 4x^2 - y^2}$.

(a) Find the domain of g.
(b) Find the range of g.

Exercise 3. Let $f(x,y) = \cos\left(\sqrt{x^2+y^2}\right)$.

(a) Plot the vertical trace of f along $x = 0$.
(b) Sketch some level curves of f.
(c) Using your answers from the previous parts, sketch the graph of f.