

Math 8 Spring 2018, Practice Exam I

Your name: _____

Instructor (please circle): ☐ Bjoern Muetzel ☐ Vardayani Ratti

INSTRUCTIONS

This is a closed book, closed notes exam.

You have 2 hours.

Use of calculators is not permitted.

(1) (**Multiple Choice**)

Which of the following vectors are perpendicular to the vector $\mathbf{v} = (1, 2, -5)$? (There may be more than one; please indicate all of them.)

- (A) $(-1, 2, 3)$ (B) $(2, -1, 0)$ (C) $(3, -4, 5)$ (D) $(3, 0, 4)$ (E) $(0, 5, 2)$

Long answer. *On each question you must show your work. No credit is given for solutions without supporting calculations. You will get partial credit for partially correct answers.*

- (2) Find the Taylor polynomial $T_3(x)$ of degree 3 centered at $a = 2$ for

$$f(x) = x^5 - 3x^2 + 5.$$

- (3) Use a Taylor polynomial for $\ln(x)$ centered at $a = 1$ to estimate $\ln(0.9)$ to within 0.01. Use the lowest degree Taylor polynomial needed to guarantee the required accuracy. Justify your answer by finding an upper bound for the remainder.

- (4) Determine whether the following series converge and explain the reason. If a series converges, compute its sum.

a) $\sum_{n=0}^{\infty} (-3)^{1-n}.$

b) $\sum_{n=2}^{\infty} \frac{5}{2^n}.$

(5) Determine the radius of convergence of the following power series.

a) $\sum_{n=3}^{\infty} \frac{n^2 + 1}{5n^2 + n} (x - 2)^n.$

b) $\sum_{n=0}^{\infty} \frac{1}{2^n} (x - 1)^{3n+1}.$

(6) Find the Maclaurin series for the following functions and determine the radius of convergence. You should give the entire series (with a formula for the general term).

a) $x^3 \cdot \exp(x^4) = x^3 \cdot e^{x^4}$.

b) $\tan^{-1}(x)$. Even if you've memorized this series, show how to obtain it from the geometric series. Be sure to justify the constant term too.

(7) Evaluate the following limit using an appropriate power series expansion.

$$\lim_{x \rightarrow 0} \frac{\cos(x^2) - 1}{3x^4}.$$

- (8) Sketch the following regions in \mathbb{R}^3 and describe them in your own words: (If you find it challenging to draw pictures in \mathbb{R}^3 , just give a very rough sketch and then a more complete description.)

a) $x = 2z$.

b) $1 \leq (x - 3)^2 + (y + 2)^2 + (z - 1)^2 \leq 4$ in \mathbb{R}^3 .

c) $x^2 + y^2 \geq 2$.

d) $y^2 = 4$.

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