# Math 8: Calculus in one and several variables
# Spring 2018 - Homework 4

Return date: Wednesday 04/25/18

**keywords:** *dot product, work, orthogonal projections, cross product*

*Instructions:* Write your answers neatly and clearly on straight-edged paper, use complete sentences and label any diagrams. Please show your work; no credit is given for solutions without work or justification.

**exercise 1.** *(4 points)* Find the acute angle between the two lines $L_1$ and $L_2$ at their point of intersection.

a) $L_1 : x + 3y = 8$ and $L_2 : 5x - 2y = 1$.

b) $L_1 : 2x - y = 5$ and $L_2 : 3x + y = 9$.

Show your work.

**exercise 2.** *(3 points)* A sled is pulled along a level path through the snow by a rope. A 80-lb force acting at an angle of 60° above the horizontal moves the sled 175 ft. Find the work done by the force.

**exercise 3.** *(2 points)* Let $\mathbf{u}$ and $\mathbf{v}$ be vectors.

a) Use the geometric interpretation of the dot product to show that

$$|\mathbf{u} \bullet \mathbf{v}| \leq |\mathbf{u}|\,|\mathbf{v}|.$$

**Note:** This inequality is called the Cauchy-Schwarz inequality.

b) Can $|\mathbf{proj}_{\mathbf{v}}(\mathbf{u})|$ be greater than $|\mathbf{u}|$? Explain your answer.

**exercise 4.** *(4 points)* Find the scalar and vector projection of $\mathbf{u}$ onto $\mathbf{v}$. Then write $\mathbf{u}$ as the sum of a vector parallel to $\mathbf{v}$ and a vector orthogonal to $\mathbf{v}$.

a) $\mathbf{u} = \langle -1, 2, 3 \rangle$ and $\mathbf{v} = \langle 3, 1, 0 \rangle$.

b) $\mathbf{u} = \langle 4, 1 \rangle$ and $\mathbf{v} = \langle 2, 3 \rangle$.

**exercise 5.** *(4 points)* For each of the following vectors, find the cross product $\mathbf{u} \times \mathbf{v}$ and verify that it is orthogonal to both $\mathbf{u}$ and $\mathbf{v}$:

a) $\mathbf{u} = \langle 1, 0, -2 \rangle$ and $\mathbf{v} = \langle -2, 1, -1 \rangle$.

b) $\mathbf{u} = t\mathbf{i} + \sin(t)\mathbf{j} + \cos(t)\mathbf{k}$ and $\mathbf{v} = \mathbf{i} - \cos(t)\mathbf{j} + \sin(t)\mathbf{k}$.

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**exercise 6.** *(3 points)*

a) Find the area of the parallelogram with vertices

$$A = (1, 1, 2)\,,\ B = (1, 2, 3)\,,\ C = (-2, 3, 0) \ \text{ and } \ D = (-2, 4, 1).$$

b) Find the area of the triangle with vertices $A$, $B$ and $C$, where $A, B$ and $C$ are the points given in part a).