Math 8: Calculus in one and several variables
Spring 2018 - Homework 3

Return date: **Wednesday 04/18/18**

keywords: *limits and integrals with Taylor series, vectors, dot product*

Instructions: Write your answers neatly and clearly on straight-edged paper, use complete sentences and label any diagrams. Please show your work; no credit is given for solutions without work or justification.

exercise 1. *(4 points)* Use Taylor series to evaluate the limit

a) $\lim_{x\to 0} \frac{\sin(x)-x-\frac{x^3}{6}}{2x^5}$.

b) $\lim_{x\to 2} \frac{\cos(x-2)-1+\frac{(x-2)^2}{2}}{(x-2)^4}$.

exercise 2. *(3 points)* Use Taylor series to evaluate $\int e^{-2x}dx$ as an infinite series.

exercise 3. *(8 points)* Describe in words the regions of $\mathbb{R}^3$ represented by the following equation(s) or inequalities. Make a simple sketch to clarify your answer.

a) $z \geq -2$.

b) $x^2 = 4$.

c) $y = z$.

d) $x^2 + y^2 = 4$ and $z = 2$.

exercise 4. *(2 points)* For the following vectors $\mathbf{u}$ and $\mathbf{v}$ calculate

$$\mathbf{u}+\mathbf{v}\,,\ 2\mathbf{u}+3\mathbf{v}\,,\ |\mathbf{u}| \ \text{ and } \ |\mathbf{u}-\mathbf{v}|$$

a) $\mathbf{u} = \langle 2, 3\rangle$ and $\mathbf{v} = \langle -2, -1\rangle$.

b) $\mathbf{u} = \langle 6, 1, -3\rangle$ and $\mathbf{v} = \langle 2, -2, 1\rangle$.

exercise 5. *(3 points)* Let $\alpha = \angle(\mathbf{u}, \mathbf{v})$ be the angle between the vectors $\mathbf{u}$ and $\mathbf{v}$. For the following vectors calculate

$$|\mathbf{u}|\,,\ |\mathbf{v}|\,,\ \frac{\mathbf{u}}{|\mathbf{u}|}\,,\ \alpha = \angle(\mathbf{u},\mathbf{v}) \ \text{ and } \ \mathbf{proj}_{\mathbf{u}}(\mathbf{v})$$

a) $\mathbf{u} = \langle 1, 4\rangle$ and $\mathbf{v} = \langle 2, 3\rangle$.

b) $\mathbf{u} = \langle -1, 4, 8\rangle$ and $\mathbf{v} = \langle 12, 1, 2\rangle$.

c) For the vectors in **5 a)** draw a picture of $\mathbf{u}$ and $\mathbf{v}$ and the projection $\mathbf{proj}_{\mathbf{u}}(\mathbf{v})$.