## Math 8: Calculus in one and several variables Spring 2018 - Homework 2

Return date: Wednesday 04/11/18

keywords: ratio test, diff. and int. of power series, Taylor series

*Instructions:* Write your answers neatly and clearly on straight-edged paper, use complete sentences and label any diagrams. Please show your work; no credit is given for solutions without work or justification.

exercise 1. (6 points) Determine whether the following series are convergent or not.

a) 
$$\sum_{n=2}^{\infty} \frac{n^3 + 3n^2 + 2n}{7n^3 + 4n + 1}$$
  
b)  $\sum_{n=10}^{\infty} \frac{(-7)^n}{n^3}$ .

c) 
$$\sum_{n=1}^{\infty} \frac{(3n)!}{(n!)^2}$$

Explain how you have obtained your answer.

exercise 2. (4 points) Determine the radius of convergence for the following series.

a) 
$$\sum_{n=0}^{\infty} \frac{n^4}{9^n} (x-7)^n$$
.  
b)  $\sum_{n=0}^{\infty} \frac{(-1)^n}{5^n \cdot n^{1/3}} (3x-2)^{2n}$ .

Explain how you have obtained your answer.

**exercise 3.** (6 points) Find power series representation for the following functions and determine the radius of convergence.

a) 
$$f(x) = \frac{2}{1-5x^3}$$
.  
b)  $f(x) = \frac{1}{(1+x)^2}$ .  
c)  $f(x) = x^2 \cdot \arctan(x^3)$ .

Justify your answer.

exercise 4. (1 points) Use the definition of Taylor and Maclaurin series to compute the terms up to degree 4 of the Maclaurin series for  $f(x) = \frac{1}{(1+x)^2}$  and compare with your answer to exercise 3b).

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**exercise 5.** (3 points) Find the sums of the following series by associating them to a Taylor series.

Hint: Look at Table 1 on page 808 of the book.

a)  $\sum_{n=0}^{\infty} \frac{x^{5n+3}}{n!}$ . b)  $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}$ .

c) 
$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$$

Justify your answer.