Math 8: Calculus in one and several variables
Spring 2018 - Homework 1

Return date: Wednesday 04/04/18

keywords: *Taylor polynomials, remainder estimate, infinite series*

Instructions: Write your answers neatly and clearly on straight-edged paper, use complete sentences and label any diagrams. Please show your work; no credit is given for solutions without work or justification.

exercise 1. *(3 points)* Find the Taylor polynomial $T_3(x)$, for the function $f(x)$ at a.

a) $f(x) = x + x^3$ at $a = 1$.

b) $f(x) = e^{2x^2+3}$ at $a = 1$.

exercise 2. *(4 points)* For each of the following problems, write out enough terms of the 100th Taylor polynomial $T_{100}(x)$, for the function $f(x)$ at the point a, to make the pattern obvious. Use whatever notation is most clear. For example, the pattern in the sequence

$$2,\ 6,\ 12,\ 20,\ 30, ...$$

becomes much easier to see if you write it as

$$1 \cdot 2,\ 2 \cdot 3,\ 3 \cdot 4,\ 4 \cdot 5,\ 5 \cdot 6, ...$$

a) $f(x) = 2e^{4x}$ at $a = 0$.

b) $f(x) = 3\ln(x + 1)$ at $a = 0$.

Explain how you have obtained your answer.

exercise 3. *(3 points)*

a) Find the Taylor polynomial $T_3(x)$, for the function

$$f(x) = x \cdot \ln(3x + 1) \quad \text{at the point} \quad a = 1.$$

b) For the values $0.6 \leq x \leq 1.4$ estimate the accuracy of the approximation using the remainder estimate

$$|R_3(x)| = |f(x) - T_3(x)|$$

in Taylor's inequality (**Theorem 11.10.9** of the book). Justify your answer.

exercise 4. *(3 points)* Suppose we use the following estimate for $3\cos(x)$:

$$3\cos(x) \simeq 3 - \frac{3}{2}x^2.$$

Explain why it's okay to estimate the error using either $R_2(x)$ or $R_3(x)$. (Note that we get a better estimate using $R_3(x)$.)

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exercise 5. *(3 points)* Determine whether the geometric series is convergent or divergent. If it is convergent, find its sum:

a) $\sum_{n=0}^{\infty} \frac{5}{(3\pi)^n}$.

b) $\sum_{n=0}^{\infty} \frac{7^{n+1}}{6^n}$.

exercise 6. *(4 points)* Find the values of x for which the series converges. Find the sum of the series for those values of x.

a) $\sum_{n=1}^{\infty} (x+7)^n$.

b) $\sum_{n=0}^{\infty} \frac{5^n}{x^n}$.