Math 8: Calculus in one and several variables
Spring 2017 - Homework 8

Return date: Wednesday 05/24/17

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keywords: *directional derivatives, gradients, extreme values*

Instructions: Write your answers neatly and clearly on straight-edged paper, use complete sentences and label any diagrams. Please show your work; no credit is given for solutions without work or justification.

exercise 1. *(3 points)* Let $f(x, y)$ be the function $f(x, y) = x^2 \cdot \ln(y)$.

a) Find the gradient $\mathbf{grad}(f)$ of f.

b) Evaluate the gradient at the point $P = (3, 1)$.

c) Find the rate of change $D_{\mathbf{u}} f(P)$ of f at P in direction from P towards the point $Q = (-2, 13)$.

exercise 2. *(2 points)* Find the maximal rate of change of the function f at the given point P and the direction in which it occurs for

a) $f(x, y) = \cos(xy)$ at $P = (0, 1)$.

b) $f(x, y, z) = x \ln(yz)$. at $P = (1, 2, \frac{1}{2})$.

exercise 3. *(4 points)* Let $f(x, y) = xy$.

a) Find the level sets $f(x, y) = k$ for $k = -1, 0$ and 1 and sketch them in the xy-plane.

b) Find the gradient $\mathbf{grad}(f)$ of f.

c) Sketch the gradient vector $\mathbf{grad}(f)(x, y)$ at several points along each of the level curves in part a). Be sure your sketch shows clearly the direction in which the gradient points and its relationship to the level set.

d) Find the tangent line to the level set $f(x, y) = -1$ at the point $P = (-2, \frac{1}{2})$.

exercise 4. *(4 points)* Find the local maxima, minima, and saddle points of the following functions.

a) $f(x, y) = 2 - x^4 + 2x^2 - y^2$.

b) $f(x, y) = (x^2 + y^2)e^x$.

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exercise 5. *(4 points)* Let D be the closed triangular region with vertices

$$A = (0,0)\ ,\ B = (0,2)\ \text{ and }\ C = (4,0).$$

Find the absolute maximum and minimum of the function

$$f(x,y) = x + y - xy \ \text{ in the region } D.$$

exercise 6. *(3 points)* Find the points on the surface

$$S:\ y^2 = 9 + xz$$

that are closest to the origin.
Hint: Instead of minimizing the distance from the origin, you can also minimize the square of the distance.