Math 8: Calculus in one and several variables
Spring 2017 - Homework 7

Return date: Wednesday 05/17/17

---

keywords: *tangent planes, chain rule in several variables*

Instructions: Write your answers neatly and clearly on straight-edged paper, use complete sentences and label any diagrams. Please show your work; no credit is given for solutions without work or justification.

exercise 1. *(3 points)* For the following functions find the indicated partial derivatives.

a) $f(x,y) = x^5y^3 - x^3y$. Find f_{xxx} and f_{xyx}.

b) $f(x,y,z) = e^{x^2+y^2+z^3}$. Find f_{xxz}.

exercise 2. *(3 points)* Find an equation for the tangent plane to the graph of the given function at the given point.

a) $f(x,y) = x^2 + 3y^2$ at the point $P = (1,1,4)$. Sketch the function and the plane.

b) $f(x,y) = e^{x+y}$ at the point $P = (2,-2,1)$. (You do not need to sketch this.)

exercise 3. *(3 points)* Let $f(x,y) = \sqrt{xy}$.

a) Find the approximation of f (i.e., the linearization) at $(x,y) = (1,4)$.

b) Use the linearization to approximate $f(0.9, 1.2)$.

exercise 4. *(4 points)* Suppose you need to know the tangent plane of a surface S containing the point $P = (2,1,3)$. You do not know the equation for S, but you know that the curves

$$\mathbf{r}_1(t) = \langle 2+3t, 1-t^2, 3-4t+t^2\rangle \quad \text{and} \quad \mathbf{r}_2(s) = \langle 1+s^2, 2s^3-1, 2s+1\rangle$$

both lie in S and pass through P.
Find an equation of the tangent plane of S passing through P.

exercise 5. *(4 points)* Use the chain rule to find $\dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial t}$.

a) $z = \arctan(x^2+y^2)$ and $x = s \cdot \ln(t)$, $y = t \cdot e^s$.

b) $z = x^2 \cdot e^{xy}$ and $x = 1 + s \cdot t$, $y = s^2 - t^2$.

Math 8: Calculus in one and several variables
Spring 2017 - Homework 7

Return date: Wednesday 05/17/17

---

exercise 6. *(3 points)* The temperature at a point (x, y) in the plane is given by the function $T(x, y)$. A bug crawls along a path c, such that its position after time t in seconds is given by

$$c: \ \mathbf{r}(t) = \langle \sqrt{1+t}, 3t - 6 \rangle, \quad \text{where} \quad t \in [0, 10].$$

From measurements you know that the temperature function satisfies

$$T_x(2, 3) = 4 \ \text{ and } \ T_y(2, 3) = 2.$$

How fast is the temperature rising on the bugs path after 3 seconds?