

**Math 8: Calculus in one and several variables**  
**Spring 2017 - Homework 6**

Return date: Wednesday 05/10/17

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**keywords:** *derivatives along curves, functions of several variables, partial derivatives*

*Instructions:* Write your answers neatly and clearly on straight-edged paper, use complete sentences and label any diagrams. Please show your work; no credit is given for solutions without work or justification.

**exercise 1.** (3 points) Consider the curve given by

$$\mathbf{r}(t) = \langle t^2, t^3 + 3, t + 1 \rangle$$

- a) Find all points on the curve for which the tangent vector is parallel to the plane  $E : 2x + y + z = 7$ .
- b) Find a parametrization of the tangent line of  $\mathbf{r}(t)$  at  $t = 1$ .
- c) Find the unit tangent vector to  $\mathbf{r}(t)$  at  $t = 1$ .

**exercise 2.** (2 points) Find the arclength  $\ell(c)$  of the curve

$$c : \mathbf{r}(t) = \left\langle \frac{t^2}{2}, \frac{2\sqrt{2}}{3}t^{3/2}, t \right\rangle \quad \text{where } 0 \leq t \leq 2.$$

**exercise 3.** (4 points) Find the domain of the following functions, then sketch the graphs and several level curves:

a)  $f(x, y) = 10 - x - y$ .

**Suggestion for graphing:** Just sketch the part of the graph in the first octant.

b)  $f(x, y) = \sqrt{4x^2 + y^2}$ .

**exercise 4.** (3 points) Consider the function  $f(x, y) = \frac{x^2 + y^2}{2x}$ . Sketch several level curves  $f(x, y) = k$ , choosing at least two positive values of  $k$  and at least two negative values of  $k$ .

**Suggestion:** After writing  $f(x, y) = k$ , clear the denominator, bring everything to one side of the equation and complete the square in  $x$ .

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**exercise 5.** (4 points) Compute the following limits or show that they do not exist.

a)  $\lim_{(x,y) \rightarrow (0,0)} \frac{e^{x^2+y^2} - 1}{x^2 + y^2}$ .

**Hint:** Look at the Maclaurin series of  $e^t$ .

b)  $\lim_{(x,y) \rightarrow (0,0)} \frac{3xy^2}{y^4 + x^2}$ .

**Hint:** Look at the curves  $\mathbf{r}_1(t) = (t, 0)$  and  $\mathbf{r}_2(t) = (t^2, t)$ .

**exercise 6.** (4 points) Find the first and second order partial derivatives of

$$f(x, y) = \cos\left(\frac{x}{2+y}\right).$$