Math 8: Calculus in one and several variables
Spring 2017 - Homework 5

Return date: Wednesday 05/03/17

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keywords: *lines, planes, space curves*

Instructions: Write your answers neatly and clearly on straight-edged paper, use complete sentences and label any diagrams. Please show your work; no credit is given for solutions without work or justification.

exercise 1. *(3 points)* Find the parametric equations and symmetric equations of the following lines. Show your work.

a) The line L_1 that passes through the points $P = (1, 0, 3)$ and $Q = (2, 1, 5)$.

b) The line of intersection of the planes $2x - y + 3z = 0$ and $x + y + 2z = 2$.
Suggestion: First find a direction vector for the line of intersection and then find one point of intersection, for example a point where $z = 0$.

exercise 2. *(3 points)* Find a (scalar) equation of the plane that passes through the points $P = (0, -2, 5)$ and $Q = (-1, 3, 1)$ and is perpendicular to the plane $E_1 : 3x + 2y + z = 10$.
Show your work.

exercise 3. *(3 points)* Find an equation for the plane consisting of all points that are equidistant for the points $P = (2, 5, 5)$ and $Q = (-6, 3, 1)$.
Explain how you have obtained your result.

exercise 4. *(4 points) (domains and limits of space curves)*

a) Find the domain of the curve $\mathbf{r}(t) = \langle \ln(6 - t), \frac{t}{t^2-1}, \sqrt{3 - t^2} \rangle$.

b) Evaluate the limit $\lim_{t \to 1} \langle \frac{\cos(3(t-1))-1}{2(t-1)^2}, \frac{t-1}{t^2-1}, \frac{1-t}{\sqrt{1-t^4}} \rangle$.

Show your work.

exercise 5. *(3 points)* Sketch the curves with the given vector equation. Indicate with an arrow the direction in which t increases.

a) $\mathbf{r}(t) = \langle \cos(t), t \rangle$ in $\mathbb{R}^2$.

b) $\mathbf{r}(t) = \langle \sin(\pi \cdot t), t, \cos(\pi \cdot t) \rangle$, $0 \leq t \leq 2\pi$, in $\mathbb{R}^3$.

exercise 6. *(4 points)* Give parametric equations for the following curves:

a) the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ in $\mathbb{R}^2$.

b) the curve of intersection of the surfaces

$$z = x^2 \quad \text{and} \quad x^2 + y^2 = 1.$$