Math 8: Calculus in one and several variables
Spring 2017 - Homework 2

Return date: Wednesday 04/12/17

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keywords: *ratio test, differentiation and integration of power series, Taylor series*

Instructions: Write your answers neatly and clearly on straight-edged paper, use complete sentences and label any diagrams. Please show your work; no credit is given for solutions without work or justification. Be sure to staple your homework.

exercise 1. *(3 points)* Determine whether the following series are convergent or not.

a) $\displaystyle\sum_{n=2}^{\infty} \frac{n^3+2n}{5n^3+1}$.

b) $\displaystyle\sum_{n=10}^{\infty} \frac{(-2)^n}{n^2}$.

Explain how you have obtained your answer.

exercise 2. *(4 points)* Determine the radius of convergence for the following series.

a) $\displaystyle\sum_{n=0}^{\infty} \frac{n^2}{3^n}(x-4)^n$.

b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n \cdot \sqrt{n}}(x-1)^{2n}$.

Explain how you have obtained your answer.

exercise 3. *(6 points)* By manipulating familiar power series (geometric series or series in **Table 1** on page 808), find power series representations centered at 0 for the following functions and determine the radius of convergence.
Do not use the ratio test to determine the radius of convergence. Instead use what you already know about the convergence properties of the series you are manipulating.

a) $f(x) = \frac{5}{1-4x^2}$.

b) $f(x) = \frac{1}{(1+x)^2}$.

c) $f(x) = x^2 \cdot \tan^{-1}(x^3)$.

Justify your answer.

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exercise 4. *(2 points)* Use the definition of Taylor and Maclaurin series to compute the terms up to degree 4 of the Maclaurin series for $f(x) = \frac{1}{(1+x)^2}$ and compare with your answer to **exercise 3b)**.

exercise 5. *(3 points)* For the function $f(x) = \cos(x)$.

a) Find the Taylor series for $f(x)$ centered at $\frac{\pi}{2}$. Find the complete series, not just the first few terms.

b) Find an upper bound for $|R_n(x)| = |f(x) - T_n(x)|$, the remainder after the nth degree Taylor polynomial and check that

$$\lim_{n\to\infty} R_n(x) = 0 \text{ for every } x.$$

Thus the Taylor series converges to the function $\cos(x)$ everywhere.

exercise 6. *(2 points)* Find the sums of the following series by associating them to a Taylor series.
Hint: Look at **Table 1** on page 808 of the book.

a) $\displaystyle\sum_{n=0}^{\infty} \frac{x^{4n+1}}{n!}$.

b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}$.

Justify your answer.