# Some Helpful Examples on Taylor Series

① Find the TSR at zero of $\sin(x^2)$

solution:

we know from class that

$$\sin x = x - x^3/3! + x^5/5! - x^7/7! + \cdots$$

So we may plug $x^2$ in on Both sides to Obtain:

$$\sin x^2 = x^2 - \frac{(x^2)^3}{3!} + \frac{(x^2)^5}{5!} - \frac{(x^2)^7}{7!} + \cdots$$

$$= x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!} + \cdots$$

② Find the TSR at 0 of $\frac{x}{1+x^2}$.

solution: we know from class that

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$$

Now, we would like to use this. Note

$$\frac{x}{1+x^2} = x \cdot \underbrace{\frac{1}{1-(-x^2)}}_{\left(\text{This is } \frac{1}{1-x} \text{ w/ } -x^2 \text{ plugged in}\right)} = x \cdot \left(1 + (-x^2) + (-x^2)^2 + (-x^2)^3 + \cdots\right)$$

$$= x\left(1 - x^2 + x^4 - x^6 + \cdots\right)$$

$$= x - x^3 + x^5 - x^7 + \cdots$$

so, the T.S.R. at 0 of $\frac{x}{1+x^2}$ is

$$x - x^3 + x^5 - x^7 + \cdots$$

$$= \sum_{n=1}^{\infty} x^{2n-1} \cdot (-1)^{n+1}$$