Math 8
Homework Set #9
Taylor Series

Practice Problems

1) The following are Taylor series (at zero) of two common functions. For each, find their intervals of convergence.

a) $\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$ b) $\arctan x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$

Find the interval of convergence for the following Taylor series:

2) $\sum_{n=0}^{\infty} \frac{x^n}{3^n \ln n}$ 3) $\sum_{n=0}^{\infty} n! x^n$ 4) $\sum_{n=0}^{\infty} \frac{(x-3)^n}{n^2+1}$

4) Find the Taylor series centered at $a = 8$ of the function $\ln x$. What is its interval of convergence?

5) Find the sum of the series

$$1 - \ln 2 + \frac{(\ln 2)^2}{2!} - \frac{(\ln 2)^3}{3!} + \frac{(\ln 2)^4}{4!} - \frac{(\ln 2)^5}{5!} + \cdots .$$

Hint: What is the Taylor series for e^x?

6) Using the Taylor series to determine the following limits:

a) $\lim_{x \to 0} \frac{\cos x - 1}{x}$ b) $\lim_{x \to 0} \frac{\arctan x - x}{x^3}$

Note: The Taylor series for $\cos x$ and $\arctan x$ are given in problem 1.

7) Use Taylor series to approximate the following integral to within 1/1000 of its exact value:

$$\int_0^1 \cos(x^2) dx.$$

Problems to Turn In

1) Suppose that $\lim_{n\to\infty} \sqrt[n]{c_n} = \dfrac{1}{R}$ so that the radius of convergence of the Taylor series $\sum_{n=1}^{\infty} c_n x^n$ is R. Find the radius of convergence of the Taylor series $\sum_{n=1}^{\infty} c_n x^{2n}$.