1) a) $f(x,y) = xe^y - ye^x$ at $(1,0)$ with $\vec{v} = \langle 3,4 \rangle$

$f_x = e^y - ye^x \quad f_x(1,0) = 1$

$f_y = xe^y - e^x \quad f_y(1,0) = 1 - e$

$\vec{u} = \frac{\vec{v}}{|\vec{v}|} = \frac{\langle 3,4 \rangle}{\sqrt{3^2+4^2}} = \langle \frac{3}{5}, \frac{4}{5} \rangle$

$f_{\vec{u}}(1,0) = D_{\vec{u}} f(1,0) = \frac{3}{5}(1) + \frac{4}{5}(1-e) = \frac{7-4e}{5}$

b) $f(x,y) = \frac{2x}{x-y}$ at $(1,3)$ with $\vec{v} = \langle 1, \sqrt{3} \rangle$

$f_x = \frac{2}{x-y} - \frac{2x}{(x-y)^2} \quad f_x(1,3) = \frac{2}{-2} - \frac{2}{(-2)^2} = -\frac{3}{2}$

$f_y = \frac{2x}{(x-y)^2} \quad f_y(1,3) = \frac{2}{(-2)^2} = \frac{1}{2}$

$\vec{u} = \frac{\vec{v}}{|\vec{v}|} = \frac{\langle 1, \sqrt{3} \rangle}{\sqrt{1^2+3}} = \langle \frac{1}{2}, \sqrt{3}/2 \rangle$

$f_{\vec{u}}(1,3) = D_{\vec{u}} f(1,3) = \left(-\frac{3}{2}\right)(1/2) + \frac{1}{2}\frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}-3}{4}$

c) $f(x,y) = y\cos(x) + x\sin(y)$ at $(0, 3\pi/2)$ with $\vec{v} = \langle 7,-2 \rangle$

$f_x = -y\sin(x) + \sin(y) \quad f_x(0, \frac{3\pi}{2}) = -\frac{3\pi}{2}(0) + (-1) = -1$

$f_y = \cos(x) + x\cos(y) \quad f_y(0, \frac{3\pi}{2}) = 1 + 0(0) = 1$

$\vec{u} = \frac{\vec{v}}{|\vec{v}|} = \frac{\langle 7,-2 \rangle}{\sqrt{49+4}} = \langle \frac{7}{\sqrt{53}}, \frac{-2}{\sqrt{53}} \rangle$

$f_{\vec{u}}(0, \frac{3\pi}{2}) = D_{\vec{u}}(0, \frac{3\pi}{2}) = -1\left(\frac{7}{\sqrt{53}}\right) + 1\left(\frac{-2}{\sqrt{53}}\right)$

$= -9/\sqrt{53}$

2) $f(x,y) = (x-1)ye^{xy}$ at $(1,4)$ in the direction $\langle 2-1, 5-4 \rangle = \langle 1,1 \rangle$. $\vec{u} = \frac{\langle 1,1 \rangle}{\sqrt{2}} = \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

$f_x = ye^{xy} + (x-1)y^2e^{xy} \quad f_x(1,4) = 4e^4 + 0$

$f_y = (x-1)e^{xy} + x(x-1)ye^{xy} \quad f_y(1,4) = 0 + 0$

$f_{\vec{u}}(1,4) = D_{\vec{u}} f(1,4) = 4e^4\left(\frac{1}{\sqrt{2}}\right) = 2\sqrt{2}e^4$

3) a) $f(x,y) = x^2 + 4y^2$ at $(0,1)$

$\nabla f(x,y) = 2xi + 8yj \qquad \nabla f(0,1) = 8j$

Direction of greatest increase

$\frac{\nabla f(0,1)}{|\nabla f(0,1)|} = \frac{8j}{8} = j$

Rate

$|\nabla f(0,1)| = 8$

b) $f(x,y) = \frac{y^2}{x+1}$ at $(3,1)$

$\nabla f(x,y) = \frac{-y^2}{(x+1)^2} i + \frac{2y}{x+1} j \qquad \nabla f(3,1) = \frac{-1}{16} i + \frac{1}{2} j$

Direction

$\frac{\nabla f(3,1)}{|\nabla f(3,1)|} = \frac{-\frac{1}{16} i + \frac{8}{16} j}{\sqrt{\frac{1}{256} + \frac{64}{256}}} = \frac{-1}{\sqrt{65}} i + \frac{8}{\sqrt{65}} j$

Rate $= |\nabla f(3,1)| = \sqrt{65}/16$

4) $f(x,y) = ye^{-xy}$ at $(0,2)$

$f_x = -y^2e^{-xy} \quad f_x(0,2) = -4$

$f_y = -xye^{-xy} \quad f_y(0,2) = 0$

Let $\vec{u} = \langle a,b \rangle$ with $a^2 + b^2 = 1$

If $f_{\vec{u}}(a,b) = D_{\vec{u}} f(a,b) = 1$, then $-4a = 1$

so $a = -1/4$, $b^2 = 15/16$ so $b = \pm\sqrt{15}/4$

Possible directions: $\vec{u} = \langle -1/4, \sqrt{15}/4 \rangle$ and $\vec{u} = \langle -\frac{1}{4}, -\sqrt{15}/4 \rangle$

5) $\overrightarrow{AB} = \langle 3-1, 3-3\rangle = \langle 2,0\rangle$, $\overrightarrow{AC} = \langle 1-1, 7-3\rangle = \langle 0,4\rangle$, $\overrightarrow{AD} = \langle 6-1, 15-3\rangle = \langle 5,12\rangle$

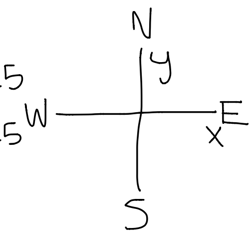
$\frac{\overrightarrow{AB}}{|\overrightarrow{AB}|} = \langle 1,0\rangle$ $\quad \frac{\overrightarrow{AC}}{|\overrightarrow{AC}|} = \langle 0,1\rangle$ $\quad \frac{\overrightarrow{AD}}{|\overrightarrow{AD}|} = \frac{\langle 5,12\rangle}{\sqrt{25+144}} = \langle \frac{5}{13}, \frac{12}{13}\rangle$

$3 = f_{\overrightarrow{AB}}(1,3) = f_x(1,3)$ $\quad 26 = f_{\overrightarrow{AC}}(1,3) = f_y(1,3)$ $\quad f_{\overrightarrow{AD}}(1,3) = 3\frac{5}{13} + 26\frac{12}{13} = 25\,^2\!/_{13}$

6) $z = 1000 - \frac{x^2}{100} - \frac{y^2}{50}$ $\qquad \frac{\partial z}{\partial x} = -x/50 \quad \frac{\partial z}{\partial y} = -y/25$

at $(10,6)$ $\qquad -1/5 \qquad -6/25$

$= -5/25$



a) Due south, will ascend
rate $6/25$

b) Due northwest, will descend.
rate $\frac{1}{5}\left(\frac{1}{\sqrt{2}}\right) - \frac{6}{25}\left(\frac{1}{\sqrt{2}}\right) = \frac{-1}{25\sqrt{2}}$

This change is small, and for other NW directions could be positive

c) Expect rate of greatest increase in a SW direction, more south than west

$\nabla z(10,6) = -\frac{1}{5}i - \frac{6}{25}j$ so greatest increase is $\frac{\langle -1/5, -6/25\rangle}{\sqrt{\frac{25}{625} + \frac{36}{625}}} = \langle \underset{W}{\frac{-5}{\sqrt{61}}}, \underset{S}{\frac{-6}{\sqrt{61}}}\rangle$

rate is $\sqrt{\frac{25}{625} + \frac{36}{625}} = \frac{\sqrt{61}}{25}$