Math 8
Homework Set #8
Construction of Taylor Series

Practice Problems

Use differentiation to find the Taylor series representation at 0 of the following functions:

1) $f(x) = \ln(1 - x)$

2) $f(x) = \cos x$

3) Use your answer from Problem 2 to find the Taylor series representation at 0 of

$$f(x) = \cos(\sqrt{x}).$$

In class we showed that

$$\frac{1}{1-x} = 1 + x + x^2 + \cdots, \tag{1}$$

which we will call equation (1). We then used this fact to find the Taylor series for $f(x) = \ln(1-x)$. In fact, we can actually use equation (1) to show a lot more as we will demonstrate through the next couple of problems.

Use equation (1) to find the Taylor series representation at 0 of the following functions

4) $f(x) = \dfrac{1}{1 + 2x^3}$

5) $f(x) = \dfrac{1}{3 + x}$

6) This problem will take you through the steps needed to find the Taylor series representation at 0 of $f(x) = \arctan(x)$.

a. Use equation (1) to find the Taylor series representation at 0 of $\dfrac{1}{1 + x^2}$.

b. Use part a) and the fact that

$$\arctan x = \int \frac{1}{1 + x^2} dx$$

to find the Taylor series representation at 0 of $\arctan x$.

Problems to Turn In

1) Use differentiation to find the Taylor series representation at 0 of

$$f(x) = \sqrt{x+1}.$$

2)

a. Using equation (1) and the fact that

$$\frac{d}{dx}\left(\frac{1}{1+x}\right) = -\frac{1}{(1+x)^2}$$

to find the Taylor series representation at 0 of $f(x) = \dfrac{1}{(1+x)^2}$.

b. Using your answer from part a. to find the Taylor series representation at 0 of

$$f(x) = \frac{1}{(1+x)^3}.$$