1) a) $z = x^2 + y^5 + 2x^2y$ $\quad x = \ln(t),\ y = t^2 + t + 1$

$\frac{\partial z}{\partial x} = 2x + 4xy$ $\quad \frac{dx}{dt} = \frac{1}{t}$ $\quad \frac{dy}{dt} = 2t + 1$

$\frac{\partial z}{\partial y} = 5y^4 + 2x^2$

$\frac{dz}{dt} = (2x + 4xy)\frac{1}{t} + (5y^4 + 2x^2)(2t+1)$

$= \left(2\ln(t) + 4\ln(t)(t^2+t+1)\right)\frac{1}{t} + \left(5(t^2+t+1)^4 + 2\ln(t)^2\right)(2t+1)$

b) $z = x^2 \cdot y^2$ $\quad x = \cos(t),\ y = \sin(t)$

$\frac{\partial z}{\partial x} = 2xy^2$ $\quad \frac{dx}{dt} = -\sin(t)$

$\frac{\partial z}{\partial y} = 2x^2y$ $\quad \frac{dy}{dt} = \cos(t)$

$\frac{dz}{dt} = 2xy^2(-\sin(t)) + 2x^2y\cos(t)$

$= -2\cos(t)\sin^3(t) + 2\cos^3(t)\sin(t)$

$= 2\cos(t)\sin(t)\left(\cos^2(t) - \sin^2(t)\right)$

2) a) $z = \sqrt{x+y}$ $\quad x = s + st + t,\ y = s\sqrt{t}$

$\frac{dz}{ds}(1,2) = \frac{1}{2\sqrt{5+\sqrt{2}}}\left(3 + \sqrt{2}\right)$

$\frac{dz}{dt}(1,2) = \frac{1}{2\sqrt{5+\sqrt{2}}}\left(2 + \frac{1}{2\sqrt{2}}\right)$

Evaluated at $(s,t) = (1,2)$

$\frac{\partial z}{\partial x} = \frac{1}{2\sqrt{x+y}} = \frac{\partial z}{\partial y}$ $\quad x = 5,\ y = \sqrt{2}$

$\frac{\partial x}{\partial s} = t + 1$ $\quad \frac{\partial x}{\partial t} = s + 1$ $\quad \frac{dx}{ds} = 3$, $\frac{dx}{dt} = 2$

$\frac{\partial y}{\partial s} = \sqrt{t}$ $\quad \frac{\partial y}{\partial t} = \frac{s}{2\sqrt{t}}$ $\quad \frac{dy}{ds} = \sqrt{2}$, $\frac{dy}{dt} = \frac{1}{2\sqrt{2}}$

b) $z = x^2 + xy + y^2$ $\quad x = r\cos(\theta),\ y = r\sin(\theta)$

$\frac{dz}{dr}(3, \pi/2) = 3(0) + 6(1) = 6$

$\frac{dz}{d\theta}(3, \pi/2) = 3(-3) + 6(0) = -9$

Evaluated at $(r, \theta) = (3, \pi/2)$

$\frac{\partial z}{\partial x} = 2x + y$ $\quad \frac{\partial z}{\partial y} = x + 2y$ $\quad x = 0$ $\quad y = 3$

$\frac{\partial x}{\partial r} = \cos(\theta)$ $\quad \frac{\partial x}{\partial \theta} = -r\sin(\theta)$ $\quad \frac{dx}{dr} = 0$ $\quad \frac{dx}{d\theta} = -3$

$\frac{\partial y}{\partial r} = \sin(\theta)$ $\quad \frac{\partial y}{\partial \theta} = r\cos(\theta)$ $\quad \frac{dy}{dr} = 1$ $\quad \frac{dy}{d\theta} = 0$

3) $V(r,h) = \frac{\pi r^2 h}{3}$

$\frac{\partial V}{\partial r} = \frac{2\pi rh}{3}$ $\quad \frac{\partial V}{\partial h} = \frac{\pi r^2}{3}$

$\frac{dr}{dt} = 2\ m/s$ $\quad \frac{dh}{dt} = -4\ m/s$

so when $r = 10,\ h = 7$

$\frac{dV}{dt} = \frac{2\pi}{3} \cdot 70 \cdot 2 - \frac{\pi}{3} \cdot 100 \cdot 4$

$= -40\pi$