1) a) $f(x,y) = (2x+3y)^{10}$

$f_x = 20(2x+3y)^9$

$f_y = 30(2x+3y)^9$

b) $f(x,y) = \dfrac{e^{-x}}{(x+y^2)}$

$f_x = \dfrac{-e^{-x}}{(x+y^2)} - \dfrac{e^{-x}}{(x+y^2)^2}$

$f_y = \dfrac{-2ye^{-x}}{(x+y^2)^2}$

c) $f(x,y) = \arctan(x\sqrt{y})$

$f_x = \dfrac{\sqrt{y}}{1+x^2y}$

$f_y = \dfrac{\frac{x}{2}y^{-1/2}}{1+x^2y}$

2) $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy\cos(y)}{3x^2+y^2}$ does not exist:

$\lim\limits_{(x,0)\to(0,0)} \dfrac{xy\cos(y)}{3x^2+y^2} = \lim\limits_{x\to 0} \dfrac{0}{3x^2} = \underline{\underline{0}}$ and $\lim\limits_{(x,x)\to(0,0)} \dfrac{xy\cos(y)}{3x^2+y^2} = \lim\limits_{x\to 0} \dfrac{x^2\cos(x)}{4x^2} = \lim\limits_{x\to 0} \dfrac{\cos(x)}{4}$

so $y=0$ so $y=x$ $= \underline{\underline{1/4}}$

3) Recall $f_{xy} = f_{yx}$.

We have $f_y = xe^{xy}\sin(y) + e^{xy}\cos(y)$, so $f_{yx} = e^{xy}\sin(y) + xye^{xy}\sin(y) + ye^{xy}\cos(y)$

4) $f(x,y) = \dfrac{x}{x+y}$ at $(2,1)$ $f(2,1) = \dfrac{2}{3}$

$f_x = \dfrac{1}{x+y} - \dfrac{x}{(x+y)^2}$, $f_y = \dfrac{-x}{(x+y)^2}$

The equation of the tangent plane is

$z - \frac{2}{3} = \left(\frac{1}{3} - \frac{2}{9}\right)(x-2) - \left(\frac{2}{9}\right)(y-1)$

$\Rightarrow$ $\underline{z - \frac{2}{3} = \frac{1}{9}(x-2) - \frac{2}{9}(y-1)}$

5) Let $f(x,y) = x^2 - xy + 3y^2$. Estimate $f(2.96, -.95)$

We find the tangent plane of f at $(3,-1)$: $z - 15 = 7(x-3) - 9(y+1)$

$f_x = 2x - y$ $f_y = -x + 6y$ $f(3,-1) = 9+3+3 = 15$ so $z = 7(x-3) - 9(y+1) + 15$

$f_x(3,-1) = 7$ $f_y(3,-1) = -9$

Evaluate the tangent plane at $(2.96, -.95)$

$z = 7(-.04) - 9(.05) + 15$ The actual answer is 14.2811

$= -.28 - .45 + 15$

$= 14.27$