1) a) $\vec{r}(t) = \langle 6+t, -5+3t, 2-4t \rangle$ or $x = 6+t$, $y = -5+3t$, $z = 2-4t$

b) $\vec{r}(t) = \langle 2t, 14-3t, -3+9t \rangle$ or $x = 2t$, $y = 14-3t$, $z = -3+9t$

c) $\vec{r}(t) = \langle t, 7t, -2t \rangle$ or $x = t$, $y = 7t$, $z = -2t$

2) The vectors for each line are $\langle -2-(-4), 0-(-6), -3-1 \rangle = \langle 2, 6, -4 \rangle = 2\langle 1, 3, -2 \rangle$
and $\langle 10-5, 18-3, 4-14 \rangle = \langle 5, 15, -10 \rangle = 5\langle 1, 3, -2 \rangle$

The direction of each is the same, so the lines are parallel, even if they are not the same. As an aside, the vector connecting $(-2,0,3)$ to $(5,3,14)$ is $\langle 7, 3, 17 \rangle$, which is a different direction, so the lines are not the same.

3) The vectors of each line are $\langle 1-(-2), 1-4, 1-0 \rangle = \langle 3, -3, 1 \rangle$
and $\langle 2-2, 3-(-1), 4-(-8) \rangle = \langle 0, 4, 12 \rangle$

There dot product is $3 \cdot 0 - 3(4) + 1(12) = 0$, so they will be perpendicular if the lines share a point. The equation for each line is

$\langle 1+3t, 1-3t, 1+t \rangle$ and $\langle 2, 3+4s, 4+12s \rangle$

Solve for t, s: $1+3t = 2$, so $t = \frac{1}{3}$

Intersection must be $\langle 2, 0, 4/3 \rangle$, so $3+4s = 0$ or $s = -3/4$,
giving $\langle 2, 0, 4 - 3/4(12) \rangle = \langle 2, 0, -5 \rangle$

The lines do <u>not</u> intersect, so they are not perpendicular.

4) a. True b. False (skew) c. true d. false e. true

5) $(x+1) + 4(y-2) - (z-4) = 0$ or $x + 4y - z = 3$

6) Two vectors in the plane are $\vec{a} = \langle 1, -2, 0 \rangle$, $\vec{b} = \langle 1, 0, -3 \rangle$, so the normal vector is

$\vec{n} = \vec{a} \times \vec{b} = 6i + 3j + 2k$, have $6(x-1) + 3y + 2z = 0$

$$\begin{matrix} i & j & k \\ 1 & -2 & 0 \\ 1 & 0 & -3 \end{matrix}$$

or $6x + 3y + 2z = 6$

7) $\vec{n_1}=\langle 1,1,-1\rangle$, $\vec{n_2}=\langle 2,-1,3\rangle$ are the normal vectors of the two planes.

Our plane must contain the line at the intersection of the planes, which has direction $\vec{n_1}\times\vec{n_2}=2i-5j-4k$ and contains a point (x,y,z) with

$$\begin{array}{ccc} i & j & k \\ 1 & 1 & -1 \\ 2 & -1 & 3 \end{array}$$

$x+y-z=2$ and $2x-y+3z-8=0$

Set $x=0$. Then $y-z=2$ $\quad -y+3z-8=0$

$y=z+2$ $\quad -z-2+3z-8=0$

so $(0,7,5)$ lies on both planes.

Our plane also contains $(-1,2,1)$, so $\langle 0-(-1),7-2,5-(-1)\rangle=\langle 1,5,6\rangle$ is a direction in our plane.

Then $\langle 2,-5,-4\rangle\times\langle 1,5,6\rangle=-10i-16j+15k$ is normal to our plane,

$$\begin{array}{ccc} i & j & k \\ 2 & -5 & -4 \\ 1 & 5 & 6 \end{array}$$

so we have $-10(x+1)-16(y-2)+10(z+1)=0$

or $-10x-16y+10z=-32$

8) Our plane is perpendicular to $\langle 2,1,-2\rangle$ and $\langle 1,0,3\rangle$, so its normal is

$\vec{n}=\langle 2,1,-2\rangle\times\langle 1,0,3\rangle=3i-8j-k$

$$\begin{array}{ccc} i & j & k \\ 2 & 1 & -2 \\ 1 & 0 & 3 \end{array}$$

Have $3(x-1)-8(y-5)-(z-1)=0$

or $3x-8y-z=-38$

9) Note the planes have normal vectors $\langle 1,1,1\rangle$ and $\langle 1,2,2\rangle$

a) Both planes contain $(1,0,0)$. The direction of the intersection is $\langle 1,1,1\rangle\times\langle 1,2,2\rangle=0i-j+k$, so the line of intersection is

$$\begin{array}{ccc} i & j & k \\ 1 & 1 & 1 \\ 1 & 2 & 2 \end{array}$$

$\vec{r}(t)=\langle 1,-t,t\rangle$ or $x=1$, $y=-t$, $z=t$

b) The angle of intersection is the angle between $\underset{\vec{a}}{\langle 1,1,1\rangle}$ and $\underset{\vec{b}}{\langle 1,2,2\rangle}$

Recall $\cos(\theta)=\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}=\frac{5}{\sqrt{3}\cdot 3}$, so $\theta=\cos^{-1}\left(\frac{5}{3\sqrt{3}}\right)\approx .276$ radians

15.79°