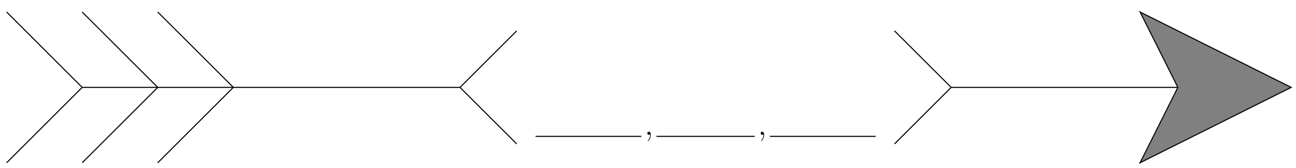
Math 8 Assignment 7

Due: Start of class on Friday May 14.

3D Vector Curve Archery

During the Famous Mathematician's Reunion of 2010 there were many events organized for the participants. One of the events was 3D vector archery. The four contestants stand at the origin $(0,0,0)$. The target is a small disk centered at the point $(0,0,2)$, that lies in the plane with normal vector $\mathbf{j}$. Each contestant is given a very special arrow:



in which they may enter any three functions they want into the arrow, and the arrow will follow that trajectory! The first contestant, Newton, enters the functions:

$$f(t) = 0 \qquad g(t) = \sin t \qquad h(t) = 1 - \cos t.$$

The second contestant, Taylor, enters the functions:

$$f(t) = \sin \frac{\pi t}{2} \qquad g(t) = \sin \frac{\pi t}{2} \qquad h(t) = t.$$

The third contestant, Simpson, enters the functions:

$$f(t) = 0 \qquad g(t) = -t^2 + 2t \qquad h(t) = t.$$

The fourth contestant, Pythagoras, enters the functions:

$$f(t) = \sin t \qquad g(t) = \cos(t) - 1 \qquad h(t) = t.$$

The job of calculus students attending the reunion (looking for hints to their exam of course) is to judge these competitions. To win the 3D vector archery competition, a contestant must hit the exact center of the target, making an angle of less than $\frac{\pi}{6}$ with the normal vector to the plane. If more than one contestant achieves this, the tie breaker will be determined by the length of the path the arrow follows, and the contestant with the shortest path wins!

Determine which contestants' arrows hit the center of the target, which arrows are at a proper angle, and if necessary determine a winner by calculating the arc length of the paths of the tied contestants.