1. (14) Find the radius of convergence and interval of convergence of

$$\sum_{n=1}^{\infty} \frac{(-1)^n (x-2)^n}{n\,5^n}$$

$$\lim_{n\to\infty} \left| \frac{(-1)^{n+1} (x-2)^{n+1}}{(n+1)\,5^{n+1}} \cdot \frac{n 5^n}{(-1)^n (x-2)^n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{x-2}{(1+1/n)\,5} \right| = \left| \frac{x-2}{5} \right|$$

By ratio test the series is cgt if $|x-2| < 5$ & dgt if $|x-2| > 5$

Hence radius of convergence $= 5$.

$-5 < x-2 < 5 \Rightarrow -3 < x < 7$

End pts: $x = -3$ $\quad \sum \frac{1}{n}$ dgt (Harmonic series)

$x = 7$ $\quad \sum \frac{(-1)^n 5^n}{n 5^n} = \sum \frac{(-1)^n}{n}$ cgt AST

Interval of cges $= (-3, 7)$

2. (10) Find a power series representation for the following function and find its interval of convergence:

$$f(x) = \frac{x}{3 - x^2}$$

$$\frac{x}{3-x^2} = x\left(\frac{1}{3-x^2}\right)$$

$$= x\left(\frac{1}{3\left(1-x^2/3\right)}\right)$$

$$= \frac{x}{3}\left(\sum_{n=0}^{\infty}\left(\frac{x^2}{3}\right)^n\right) \quad \text{if } \left|\frac{x^2}{3}\right| < 1$$

$$= \sum_{n=0}^{\infty} \frac{x^{2n+1}}{3^{n+1}}$$

if $|x| < \sqrt{3}$ series cges & if $|x| \geq \sqrt{3}$

series is dgt

Interval $= (-\sqrt{3}, \sqrt{3})$

3. (14) Find the first 2 nonzero terms in the Maclaurin series for $f(x) = \sec x$.

$f(x) = \frac{1}{\cos x}$ $\qquad$ $f'(x) = \sec x \tan x$

$f(0) = 1$ $\qquad$ $f'(0) = 0$

$f''(x) = \sec x(\sec^2 x) + \tan x(\sec x \tan x)$

$f''(0) = 1$

$1 + \frac{1x^2}{2!} = 1 + \frac{x^2}{2}$

4. (10) Find a vector that has the same direction as $\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ but has length 3.

$\vec{i} + \vec{j} + 2\vec{k} = \langle 1, 1, 2 \rangle$

If the vector is $\langle t, t, 2t \rangle$

Then $\sqrt{t^2 + t^2 + 4t^2} = 3$

$\Rightarrow \sqrt{6t^2} = 3$

$\Rightarrow t^2 = 3/2$

$\Rightarrow t = \pm\sqrt{3/2}$.

There vector is $\left\langle \frac{\sqrt{3}}{2}, \sqrt{3}/2, 2\sqrt{3/2} \right\rangle$

5. (10) Find the scalar and vector projections of $\mathbf{b} = \langle 3, 0, 2 \rangle$ onto $\mathbf{a} = \langle -2, 1, -1 \rangle$.

scalar projection of $\vec{b}$ onto $\vec{a}$ is

$$\frac{\vec{b} \cdot \vec{a}}{|\vec{a}|}$$

$$= \frac{-6-2}{\sqrt{4+1+1}} = \frac{-8}{\sqrt{6}}$$

vector proj'n of $\vec{b}$ onto $\vec{a}$ $= \frac{b \cdot a}{|a|^2} \vec{a}$

$$= -\frac{8}{\sqrt{6}} \frac{\langle -2, 1, -1 \rangle}{\sqrt{6}}$$

$$= -8/6 \langle -2, 1, -1 \rangle$$

$$= \langle 8/3, \frac{-4}{3}, 4/3 \rangle$$

6. (12) Find parametric equations for the line through the point $(0, 14, -10)$ and parallel to the line $x = -1 + 2t, y = 6 - 3t, z = 3 + 9t$

parallel vector $= \langle 2, -3, 9 \rangle$

$x_0 = (0, 14, -10)$

parametric eq^n^s

$x = 0 + 2t = 2t$

$y = 14 - 3t$

$z = -10 + 9t$

7. (14) Let P be the plane passing through $A = (0,1,1), B = (2,-1,3)$ and $C = (1,1,-2)$. Find an equation of the plane passing through $(4,-2,3)$ and parallel to the plane P. Write the equation in the form of $ax + by + cz = d$.

$$\vec{AB} = \langle 2, -2, 2 \rangle$$

$$\vec{AC} = \langle 1, 0, -3 \rangle$$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} i & j & k \\ 2 & -2 & 2 \\ 1 & 0 & -3 \end{vmatrix}$$

$$= \langle 6, 8, 2 \rangle$$

normal vector to P is $\langle 6, 8, 2 \rangle$

eqn of the plane

$$6(x-4) + 8(y+2) + 2(z-3) = 0$$

$$6x + 8y + 2z = 14$$

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8. (16) For each of the following statements, fill in the blank with the letters **T** or **F** depending on whether the statement is true or false. You do not need to show your work and no partial credit will be given on this problem.

(a) $e^3 = \sum_{n=0}^{\infty} \frac{3^n}{3!}$

$$e^3 = \sum_{n=0}^{\infty} \frac{3^n}{n!}$$

ANS: F

(b) The interval of convergence for the power series of $f(x) = ln(1 + x)$ is $(-1, 1)$.

$(-1, 1]$

ANS: F

(c) Let θ be the angle between $\mathbf{a} = \langle \mathbf{1}, \mathbf{2}, \mathbf{3} \rangle$ and $\mathbf{b} = \langle -\mathbf{5}, \mathbf{1}, \mathbf{0} \rangle$, then $\theta > \frac{\pi}{2}$

$\vec{a} \cdot \vec{b} = -3$

ANS: T

(d) $|\mathbf{a} \times \mathbf{a}| = (|\mathbf{a}|)^2$.

$|\vec{a} \times \vec{a}| = 0$.

ANS: F