

### Assignment 18: Matrix equations and matrix algebra

1. For the two matrices below, find the dimension of the solution space of  $A\mathbf{x} = \mathbf{0}$ . Then find all the solutions. You should be able to determine the dimension without finding the solutions.

$$A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \text{ and } A = \begin{pmatrix} 1 & 2 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 \end{pmatrix}$$

2. Suppose that the matrix  $A$  has row-reduced echelon form  $R$  given below.

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 1 \\ 2 & 3 & 4 & 5 & 6 \\ 7 & 8 & 9 & 1 & 2 \end{pmatrix} \text{ and } R = \begin{pmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Find all solutions to the matrix equation  $A\mathbf{x} = \begin{pmatrix} 6 \\ 7 \\ 8 \\ 9 \end{pmatrix}$  given that  $\mathbf{x} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$  is a

particular solution.

3. Let  $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ ,  $B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$ , and  $C = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$ .

Determine which of the nine products  $A^2, AB, AC, BA, B^2, BC, CA, CB, C^2$  are defined. Evaluate the first three valid products.

4. Find  $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}^{1234}$ . You may want to find  $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}^2$  and  $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}^3$  to start. Can you prove your result?
5. Find two  $2 \times 2$  matrices  $A$  and  $B$  neither of which is the zero matrix, but for which  $AB = \mathbf{0}$ .