Math 8
Fall 2015

Preliminary Homework
Due Monday, September 21

Note: Preliminary homework is always graded credit or no credit. **You get full credit for completing the assignment, whether or not your answers are correct.** The purpose of preliminary homework is to start you thinking about the topic of the next class.

You may use your preliminary homework in activities with your classmates. You should be sure to think about these questions so you will be prepared.

Preliminary homework is always due at the *beginning* of class.

These questions introduce the idea of the limit of an infinite sequence of numbers. It should make intuitive sense that the sequence

$$.1, .01, .001, .0001, .00001, .000001, .0000001, \cdots$$

is approaching 0 as a limit, while the sequence

$$1, 2, 3, 4, 5, 6, 7, \cdots$$

is not approaching 0 as a limit.

1. Decide whether each of the following sequences is approaching 0 as a limit.

   (a) $1.1, 1.01, 1.001, 1.0001, 1.00001, 1.000001, 1.0000001, \cdots$

   (b) $1, .1, .01, .001, .0001, .00001, .000001, .0000001, \cdots$

   (c) $-1.1, -1.01, -1.001, -1.0001, -1.00001, -1.000001, -1.0000001, \cdots$

   (d) $1, -.1, .01, -.001, .0001, -.00001, .000001, -.0000001, \cdots$

   (e) $1, .01, .1, .0001, .001, .000001, .00001, .00000001, \cdots$

   (f) $1, .1, 1, .001, 1, .00001, 1, .0000001, \cdots$

   (g) $0, -.9, -.99, -.999, -.9999, -.99999, -.999999, -.9999999, \cdots$

2. Which of the following definitions do you think correctly defines the notion "the sequence a_1, a_2, a_3, $\cdots$ approaches 0 as a limit"? (There may be more than one correct answer.) Think about the examples in problem (1).

(a) The terms a_n are getting smaller. That is, for every n, we have $a_{n+1} < a_n$.

(b) The terms a_n are getting smaller, and if p is any positive number, then for some n we have $a_n < p$.

(c) The terms a_n are getting smaller, and if p is any positive number, then for some n we have $a_m < p$ for every $m \geq n$.

(d) If p is any positive number, then for some n we have $|a_n| < p$.

(e) The absolute values of terms $|a_n|$ are getting smaller, and if p is any positive number, then for some n we have $|a_m| < p$ for every $m \geq n$.

(f) If p is any positive number, then for infinitely many n we have $|a_n| < p$.

(g) If p is any positive number, then for some n we have $|a_m| < p$ for every $m \geq n$.

(h) If p is any positive number, then there are only finitely many n with $|a_n| \geq p$.