MATH 8 FALL 2010 MIDTERM 2 REVIEW SHEET

This sheet contains a quick summary of definitions, tests, and other facts which might be useful for the second midterm. There are no examples on this sheet, and is only intended to be a convenient reference where you can see all the basic ideas of the past four weeks in a single place. The following lists may not be comprehensive – it is possible that I forgot to include certain definitions or theorems!

1. Power series, Taylor series

- Know power series for $\sin x, \cos x, e^x, \frac{1}{1-x}, \ln(1+x), \arctan x$.
- Know Taylor's formula, for computing a Taylor series using derivatives of a function.
- Know how to manipulate power series by multiplying by a scalar, a power of x, substituting a power of x for x, differentiating, and integrating power series.

2. Integration

- The integration by parts formula is

$$\int f(x)g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx.$$

This formula also goes under the form

$$\int u\,dv = uv - \int v\,du.$$

- Integration by parts works well on expressions like $\int x^n e^x\,dx, \int x^n \cos x\,dx, \int e^x \cos x\,dx$, etc.
- Integration by parts also works on $\int \ln x\,dx, \int \arctan x\,dx$.
- A trigonometric integral is an integral in one of the three forms

$$\int \sin^n x \cos^m x\,dx, \int \tan^n x \sec^m\,dx, \int \csc^n x \cot^m x\,dx$$

where n, m are non-negative integers. There are different strategies for dealing with each of these integrals, depending on the type and whether m, n are odd or even. Consult the book for more details.
- The strategies for the above integrals are all based on the essential identities

$$\sin^2 x + \cos^2 x = 1, \tan^2 x + 1 = \sec^2 x, 1 + \cot^2 x = \csc^2 x.$$

- A trigonometric substitution is any substitution (in an integral) of the form

$$x = a\sin\theta, x = a\tan\theta, x = a\sec\theta.$$

These substitutions are useful for solving integrals which include

$$\sqrt{a^2 - x^2}, \sqrt{x^2 + a^2}, \sqrt{x^2 - a^2}$$

respectively. You will usually end up with a trigonometric integral which you then evaluate using the rules mentioned earlier.

3. Vectors, Lines and planes in $\mathbb{R}^3$.

- A vector is a point in $\mathbb{R}^n$, and is often represented as an arrow which starts at the origin and ends at the point in question. These arrows are also translated occasionally and start at points besides the origin.
- The dot product of $\vec{v} = \langle v_1, \ldots, v_n \rangle$ and $\vec{w} = \langle w_1, \ldots, w_n \rangle$ is $\vec{v} \cdot \vec{w} = v_1 w_1 + \ldots + v_n w_n$.
- The length of a vector $\vec{v} = \langle v_1, \ldots, v_n \rangle$ is $\sqrt{v_1^2 + \ldots + v_n^2}$ and is written $|\vec{v}|$ or $||\vec{v}||$. The length is also equal to $\sqrt{\vec{v} \cdot \vec{v}}$.
- Suppose θ is the angle between two vectors $\vec{v}, \vec{w}$. Then
$$\cos\theta = \frac{\vec{v} \cdot \vec{w}}{|\vec{v}||\vec{w}|}.$$
In particular, $\vec{v} \cdot \vec{w} = 0$ if $\vec{v}, \vec{w}$ are perpendicular (orthogonal, normal) to each other.
- The vector projection of $\vec{b}$ onto $\vec{a}$ is the vector
$$\frac{\vec{b} \cdot \vec{a}}{|\vec{a}|} \frac{\vec{a}}{|\vec{a}|}.$$
This should be thought of as the part of $\vec{b}$ which points in the same direction as $\vec{a}$. The scalar projection of $\vec{b}$ onto $\vec{a}$ is the scalar
$$\frac{\vec{b} \cdot \vec{a}}{|\vec{a}|}$$
which is, up to a $\pm$ sign, the length of the vector projection.
- Suppose $\vec{v} = \langle v_1, v_2, v_3 \rangle$ and $\vec{w} = \langle w_1, w_2, w_3 \rangle$ are two vectors in $\mathbb{R}^3$. Then their cross product is the vector
$$\vec{v} \times \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix} = (v_2 w_3 - v_3 w_2)\vec{i} - (v_1 w_3 - v_3 w_1)\vec{j} + (v_1 w_2 - v_2 w_1)\vec{k}.$$
where the bars denote the determinant of a 3×3 matrix.
- $\vec{v} \times \vec{w}$ is perpendicular to both $\vec{v}, \vec{w}$. $\vec{v} \times \vec{w} = \vec{0}$ if $\vec{v}, \vec{w}$ are scalar multiples of each other.
- $|\vec{v} \times \vec{w}|$, the length of the cross product, is equal to the area of the parallelogram spanned by $\vec{v}, \vec{w}$.
- The volume of the parallelepiped spanned by three vectors $\vec{a}, \vec{b}, \vec{c}$ in $\mathbb{R}^3$ is $|\vec{a} \cdot (\vec{b} \times \vec{c})|$.
- A line in $\mathbb{R}^3$ is completely determined by a direction vector $\vec{v}$, and any point on the line. If (x_0, y_0, z_0) is a point on the line, and $\langle a, b, c \rangle$ a direction vector for the line, then the line is given by the parametric equations
$$x = x_0 + at, y = y_0 + bt, z = z_0 + ct.$$
- A line can also be given by symmetric equations, which will have the form
$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}.$$
Naturally, for these equations to make sense, all of a, b, c need to be nonzero.
- You should know how to determine when a pair of lines is parallel, skew, or intersecting, and in the intersecting case, how to calculate the point of intersection.
- A plane is completely determined by a normal vector $\vec{n} = \langle a, b, c \rangle$ and a point (x_0, y_0, z_0) on the plane. In this case, the equation for a plane is given by
$$ax + by + cz = ax_0 + by_0 + cz_0.$$

We sometimes write $d = ax_0 + by_0 + cz_0$. A normal vector for a plane has the defining property that it is orthogonal to every vector which lies on the plane (that is, any vector formed by connecting any two points on the plane).

- The angle between two planes is equal to the angle between their two normal vectors, up to the stipulation that the angle between two planes is always less than or equal to $90°$.
- You should know how to calculate whether two planes intersect or not, and if they do intersect, what the equation for their line of intersection is, and also the same question for a line and a plane. You should also know how to use vector projections to calculate the distance of a point from a plane.

4. Tips for preparing

The following suggestions may or may not help you prepare for the test. (This is identical advice to that given on the last review sheet.)

- Take a look at all your old homework assignments, and learn how to solve them all by yourself. Many people either received assistance or took many attempts to correctly answer questions – at this point, you should be able to solve every old homework problem, correctly, on your own.
- For additional practice problems, try solving random questions from the text.
- There is a practice exam available on the course webpage.
- Make sure you are precise with arithmetic and algebra. Being able to do both these accurately, by hand, is an important skill to cultivate.
- Make sure you are well-rested and not hungry during the exam. You may also want to bring a light coat, in case the room is too cold. Also make sure you bring enough pencils or pens to the test site.
- This isn't a tip, but PLEASE turn off all cell phones and other portable electronic devices during the exam. Even phones on vibrate settings are a nuisance to nearby test takers.
- Try to keep the time limit in mind as you take the exam. Do not spend too much time obsessing about a small number of problems if it will be detrimental to your performance on the remaining problems.
- Show all your work, and try to be organized and neat when you write your solutions. This will make it easier for the graders to follow your line of thought, and may help you earn (more) partial credit if you cannot completely solve a question correctly.